Homework 5: Graphs and Geometry V

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Problem 1.
Show that for any positive integer n there exists a graph on $2n$ whose degrees of vertices are $1, 1, 2, 2, 3, 3, \ldots, n, n$ (this is the list of the degrees of all $2n$ vertices).

Proof. Induction on n. Base is clear. For the step, let us take this graph on $2n$ vertices. Now add two vertices and connected them together. Also connect one of the new vertices two one vertex of degree k among the old $2n$ vertices for each $1 \leq k \leq n$. It is easy to check that this new graph satisfies the conditions of the problem for $n + 1$, and we are done. □

Problem 2.
The angle $\angle A$ of a rhombus $ABCD$ is equal to $60°$. Points M and N were chosen on AB and BC respectively, in such a way that $AM = BN$. Show that the triangle MDN is equilateral.

Proof. First we prove that $\triangle DAM = \triangle DBN$ using the SAS test. Then $DM = DN$. Moreover $\angle ADM = \angle BDM$ from which we can conclude that $\angle MDN = \angle ADB = 60°$. Then $\triangle MDN$ is equilateral and we are done. □