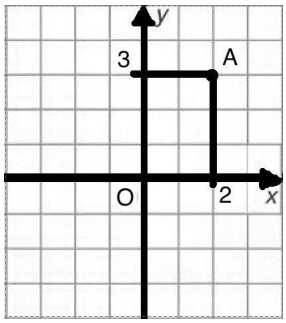
Exploring the City of Descartes-Part I

Junior Circle 04/03/32011

René Descartes was a French mathematician, philosopher and writer. Among his many accomplishments, he developed a very convenient way to describe positions of points on a plane. This method was very important for future development of mathematics and physics. We will start learning about this invention today.

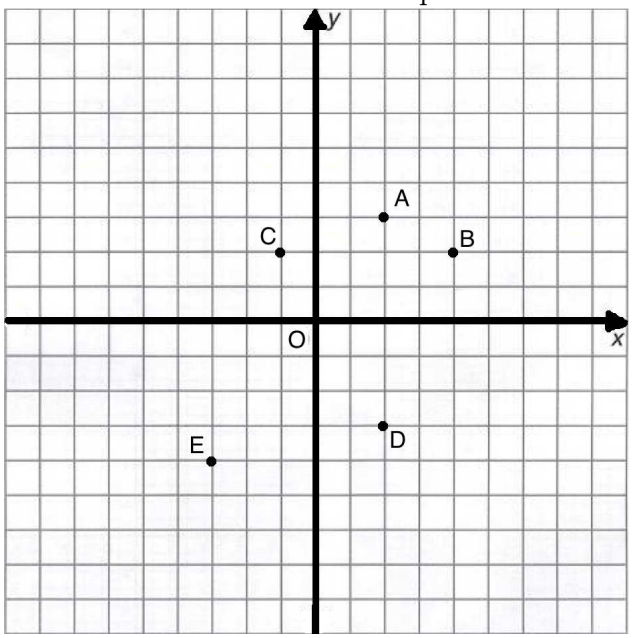
The city of Descartes is a plane that extends infinitely in all directions:

- The center of the city is marked by point O.
- The horizontal (West-East) line going through O is called the x-axis.
- The vertical (South-North) line going through O is called the y-axis.
- Each house in the city is represented by a point which is the intersection of a vertical and a horizontal line. Each house has an address which consists of two whole numbers written inside of parenthesis. For example, $(2, 3)$ is an address of the house A shown below.



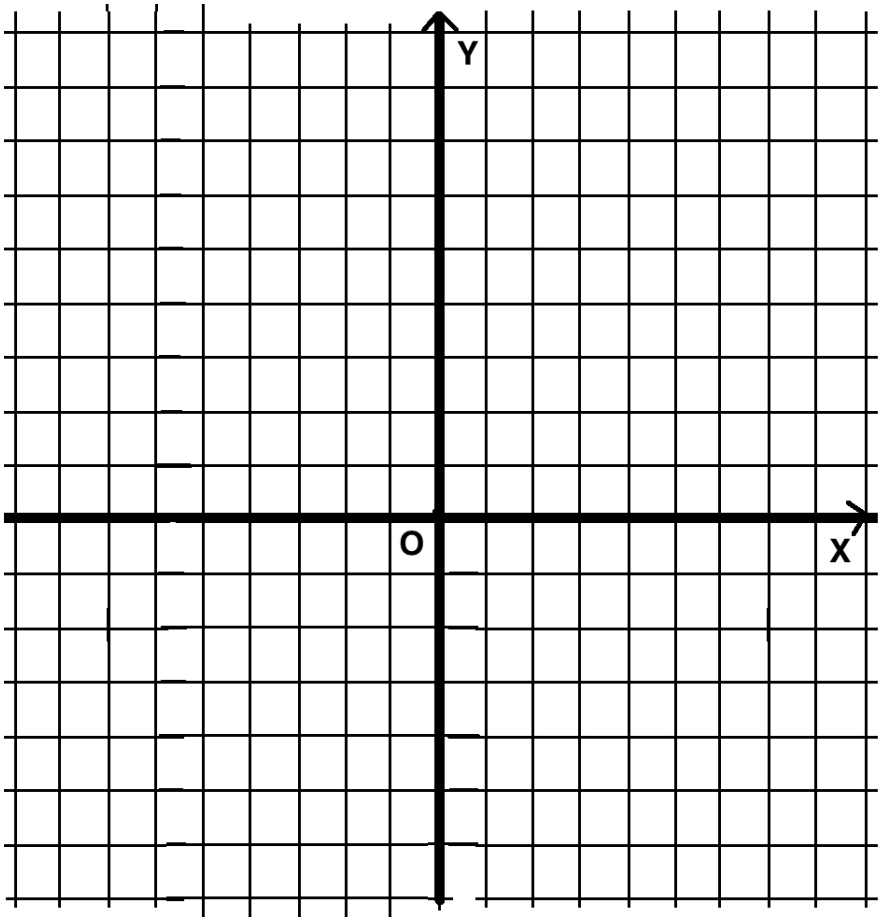
- The first number tells you the distance to the y-axis. The distance is *positive* if you are on the right of the y-axis. The distance is *negative* if you are on the left side of the y-axis.
- The second number tells you the distance to the x-axis. The distance is *positive* if you are above the x-axis. The distance is *negative* if you are below the y-axis.

1. Let's find addresses of several points in the city:



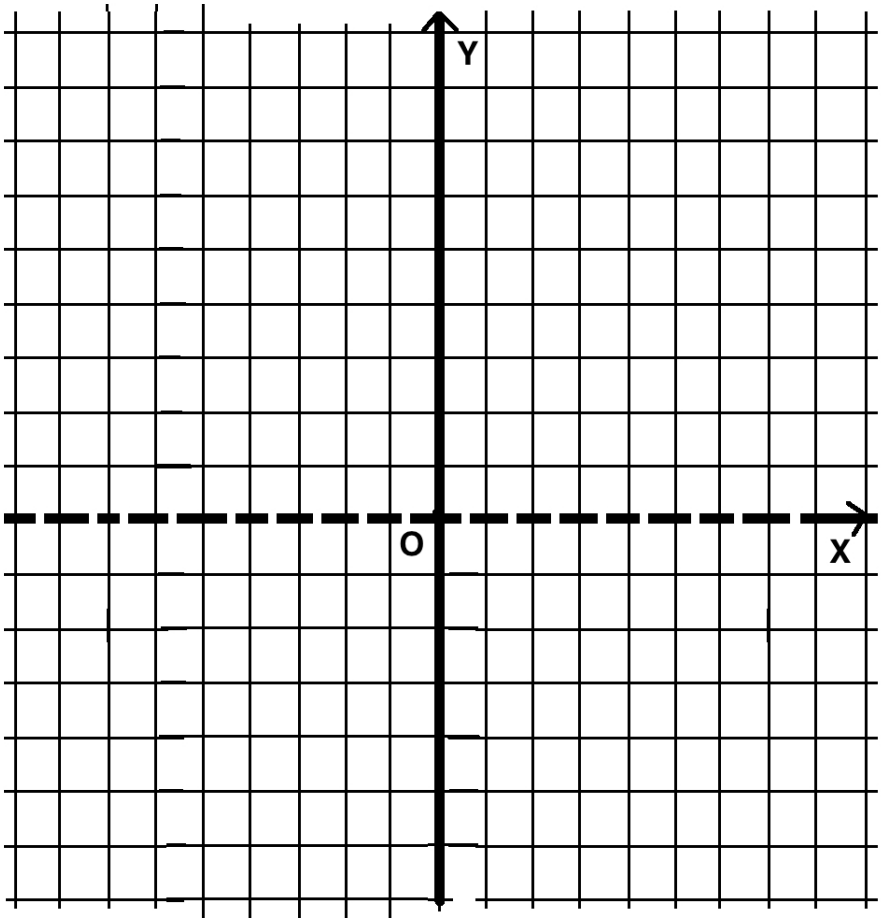
   (a) Point O has address (,);
   (b) Point A has address (,);
   (c) Point B has address (,);
   (d) Point C has address (,);
   (e) Point D has address (,);
   (f) Point E has address (,);
   (g) The midpoint between A and D has address (,);

2. Let's plot several points whose addresses are given:



(a) Plot point F with address $(1, 4)$;
(b) Plot point G with address $(4, 1)$;
(c) Plot point H with address $(5, 3)$;
(d) Plot point J with address $(2, 6)$;
(e) Plot point K with address $(0, 2)$;
(f) Plot point L with address $(3, 0)$.

3. Plot the points with addresses given below. Now imagine that the x-axis is a mirror. Reflect each of the points below and find the address of the reflection:



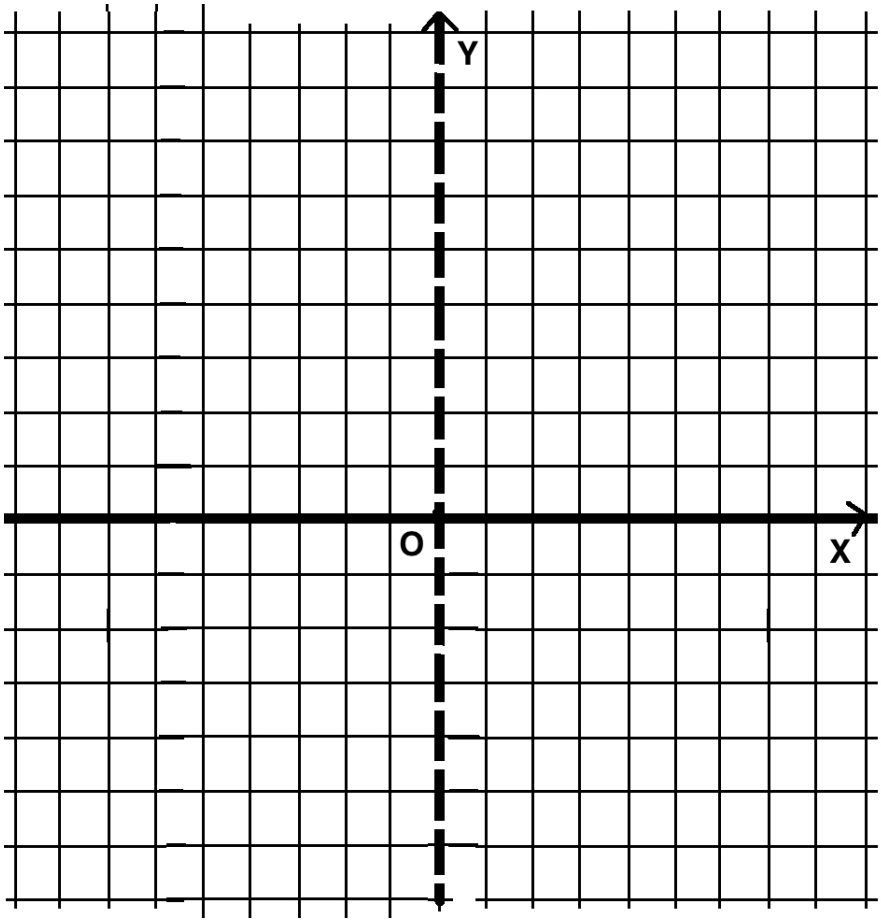
(a) $(2, 3)$

(b) $(5, 1)$

(c) $(-6, 2)$

(d) $(-3, 4)$

4. Plot the same points again. Now imagine that the y-axis is a mirror. Reflect the points below and find the address of the reflections:



(a) $(2, 3)$

(b) $(5, 1)$

(c) $(-6, 2)$

(d) $(-3, 4)$

5. Let n be any whole number. Describe where the points with the following addresses are located:

   (a) with addresses $(n, 0)$:

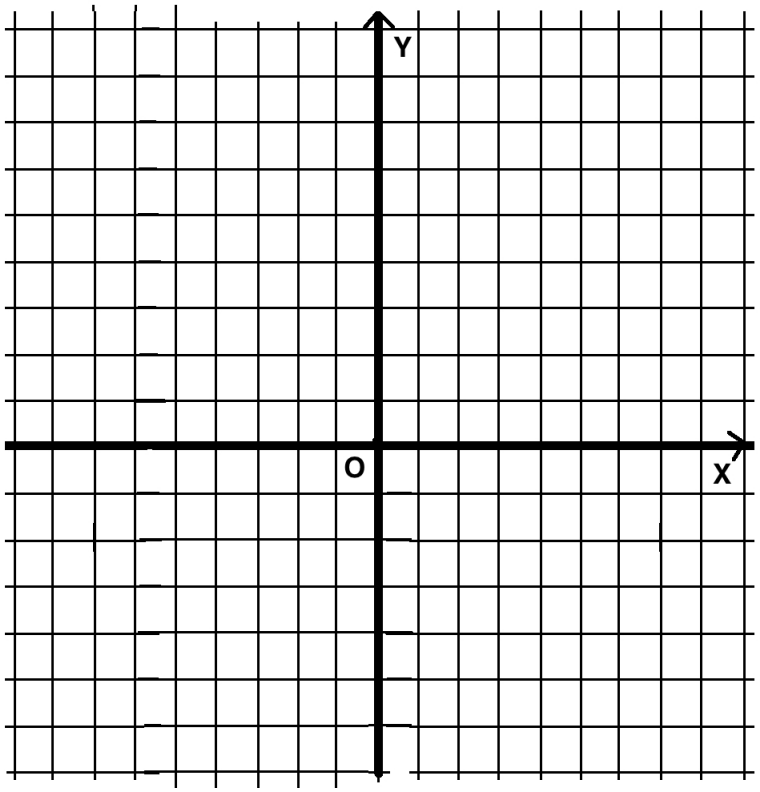
   (b) with addresses $(0, n)$:

   (c) with addresses $(n, 5)$:

   (d) with addresses $(5, n)$:

   (e) with addresses (n, n):

6. Plot the points and find the distance between the following points:



(a) $(4, 3)$ and $(4, 7)$;
Distance=

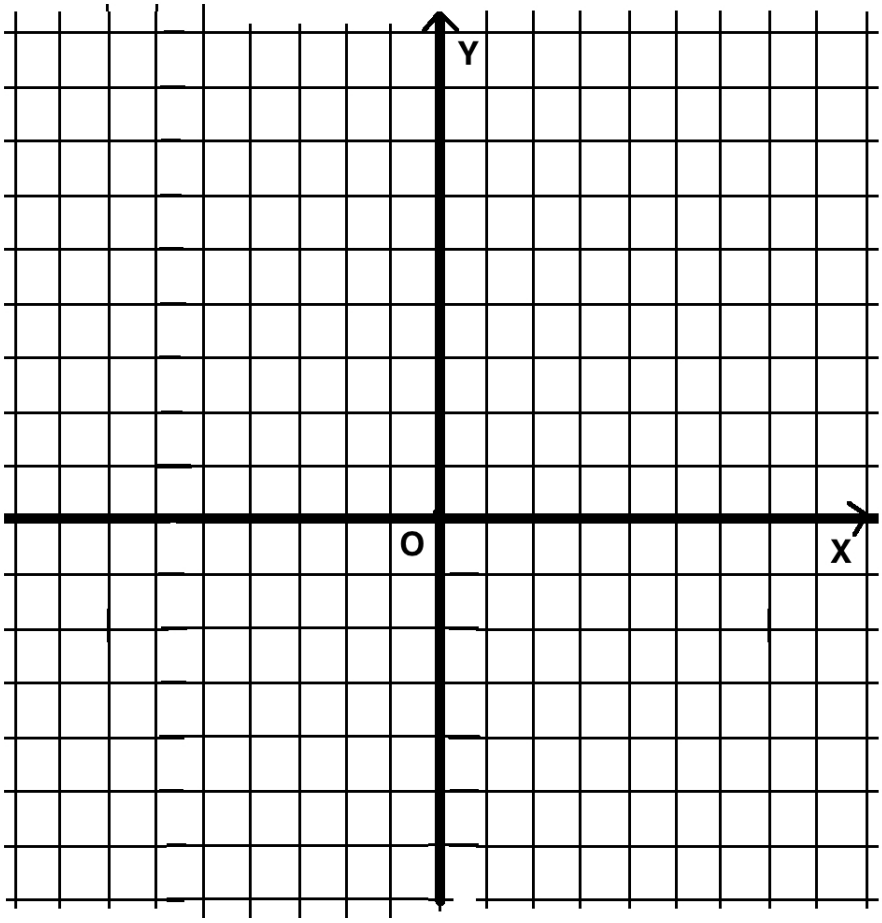
(b) $(-1, 3)$ and $(-1, 5)$;
Distance=

(c) $(6, 5)$ and $(8, 5)$;
Distance=

(d) $(5, -2)$ and $(7, -2)$;
Distance=

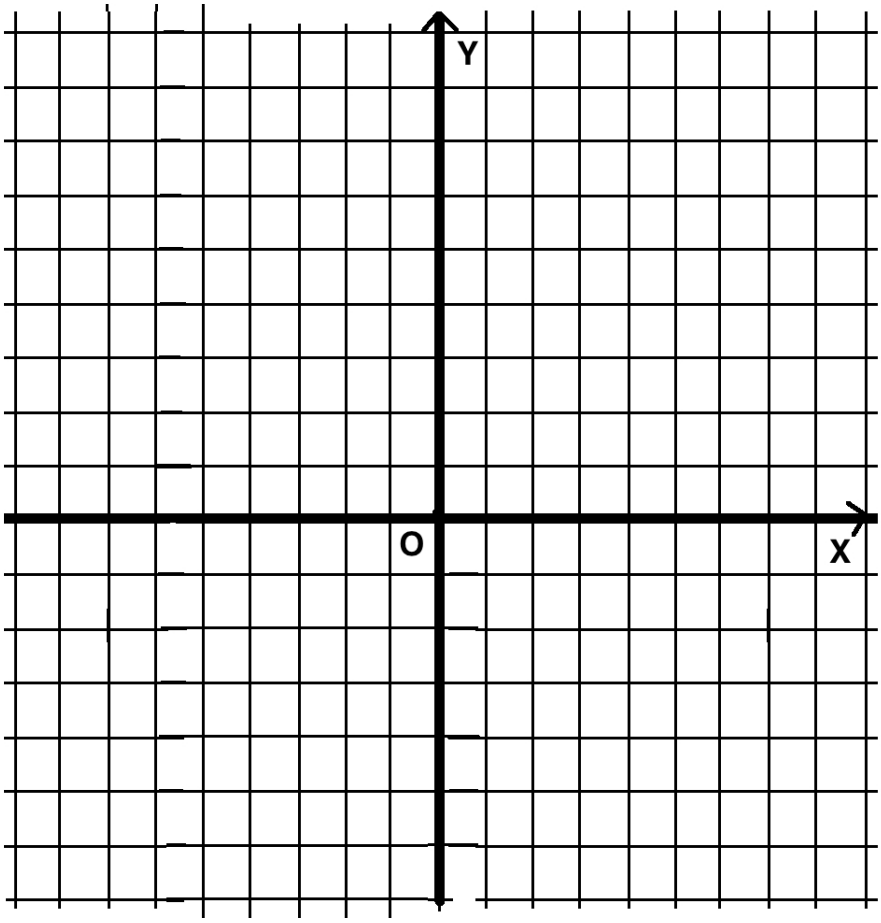
7. You start at the point $(0, 0)$. Then

- Go north (up) for two units;
- Go east (right) for 5 units;
- Go south for 1 unit;
- Go west for 1 unit.

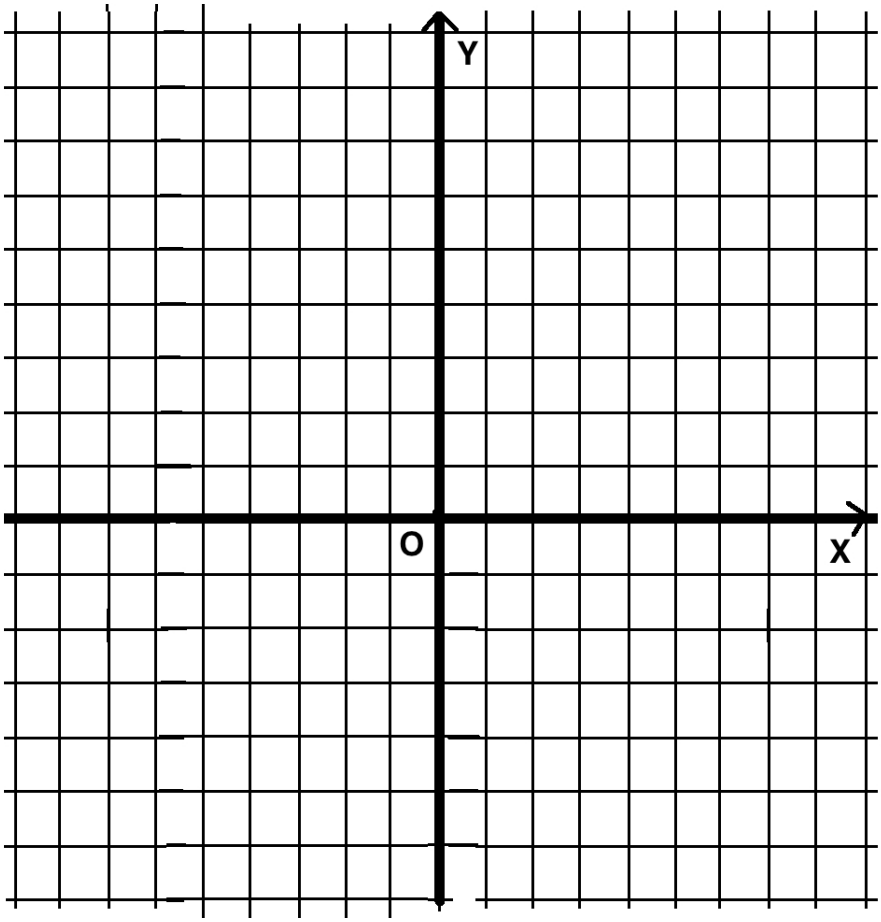


Where are you now? Give the address of the final point.

8. Two of the vertices of a square are at points $(2, 3)$ and $(5, 3)$. Find the other two vertices for the square and write down their coordinates. *Note:* The sides of the squares are vertical and horizontal. *Hint:* There is more than one solution, find all possible solutions.



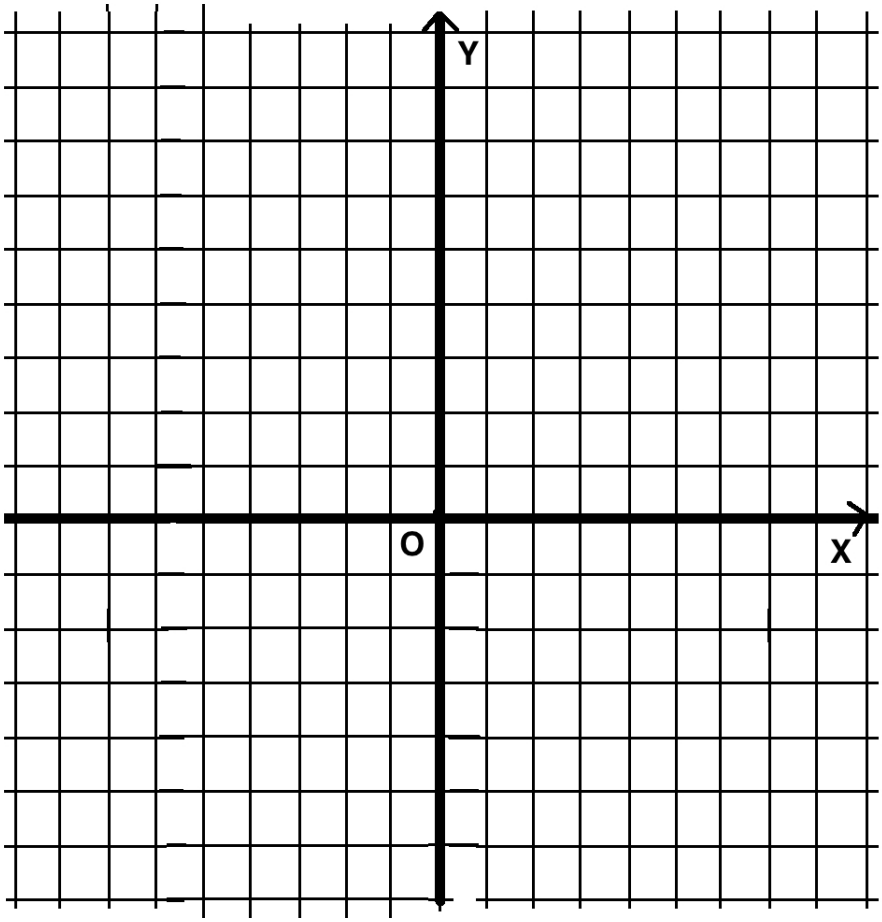
9. Two of the vertices of a square are at the points $(0, 0)$ and $(4, 4)$. Two of the sides of the square are vertical and the other two sides are horizontal. Find the other vertices of this square:



10. For the points $(2, 5)$ and $(8, 5)$

(a) Find the midpoint of the segment connecting them;

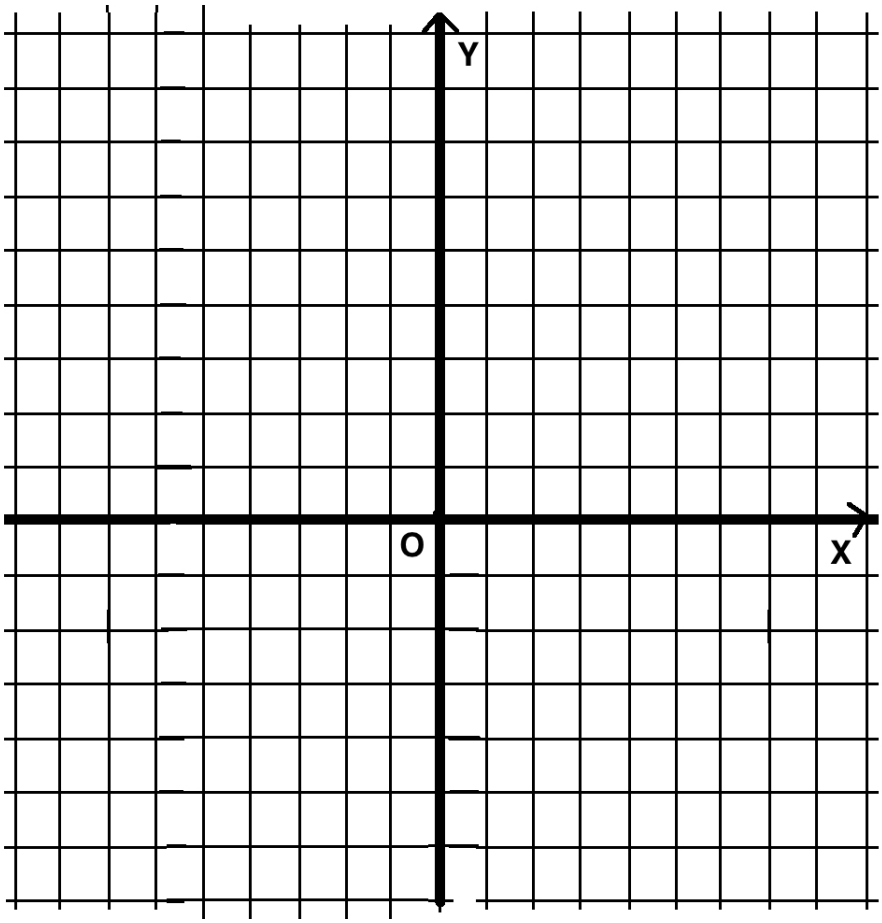
(b) Find the point which is on the x-axis and has the same distance to the points;



11. Draw two lines on the plane so that:

    (a) the first goes through $(-4, 2)$ and $(-2, 2)$;

    (b) the other goes through $(2, 6)$ and $(2, 4)$.

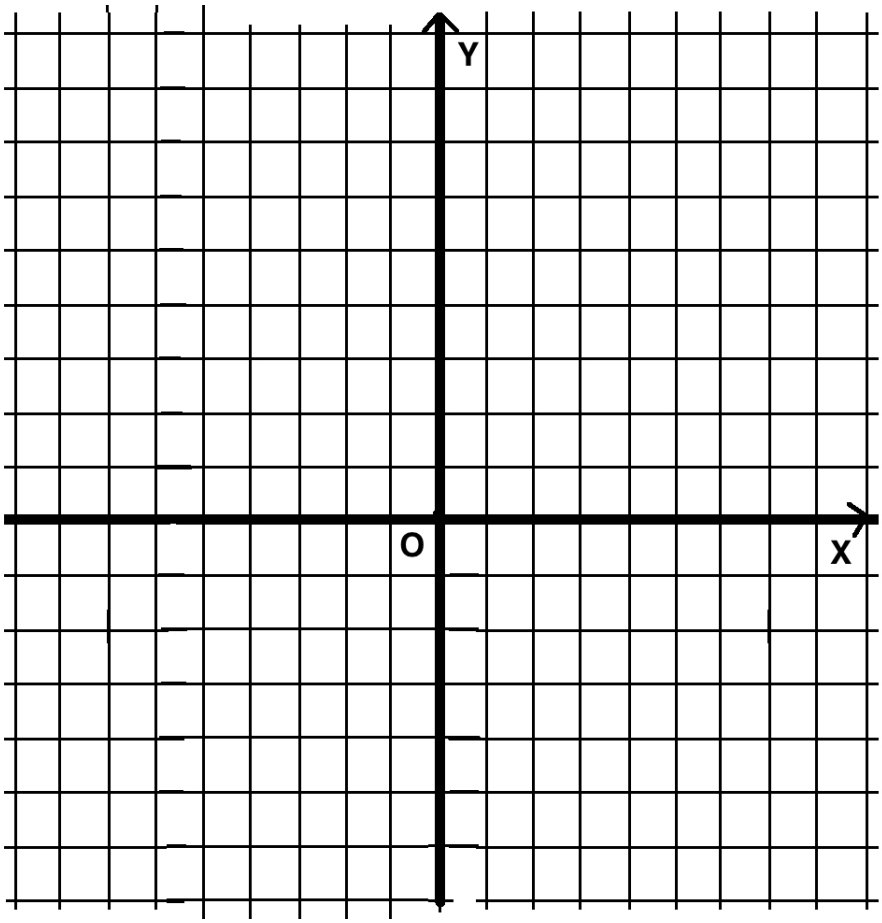


What is the point of intersection of these two lines? Give the address of this point.

12. Draw two lines on the plane so that:

(a) the first goes through $(1, 1)$ and $(4, 4)$;

(b) the other goes through $(0, 4)$ and $(4, 0)$.



What is the point of intersection of these two lines? Give the address of this point.