Lesson 7: Games and Geometry II

Konstantin Miagkov

Problem 1.

a) Two players are taking turns moving a *limp king* on a 7×7 board. A limp king is a chess piece which can move one square to the right or diagonally to the right and up. The limp king starts at the bottom-left corner of the board. The player who cannot make a turn loses. Who has a winning strategy?

b) Same problem, but now the limp king can also go up one square.

Problem 2.

There is a bunch of 10 million matches. Two players are playing the next game. They take turns. In one turn, the player can take p^n matches from the bunch, where p is a prime number and $n = 0, 1, 2, 3, \dots$ (for example, the first takes 25 matches, the second 8, the first 1, the second one 5, the first - 49, etc.). The one who takes the last match wins. Who will win if both players play optimally?

Problem 3.

Find the geometric locus of points from which the tangents drawn to a given circle are congruent to a given segment.

Problem 4.

a) Two lines passing through a point M are tangent to a circle at the points A and B. Through a point C taken on the smaller of the arcs AB, a third tangent is drawn up to its intersection points D and E with MA and MB respectively. Prove that the perimeter of $\triangle DME$ does not depend on the position of the point C.

Hint: The perimeter is congruent to $MA + MB$;

b) Prove that the angle $\angle DOE$ (where O is the center of the of circle) does not depend on the position of the point C.

Hint: $\angle DOE = \frac{\angle AOB}{2}$