PRACTICE TEST 3

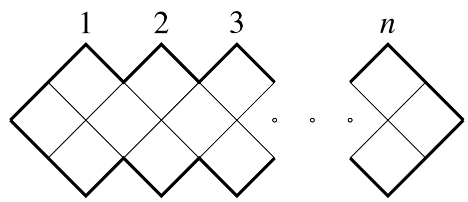
LAMC OLYMPIAD GROUP, WEEK 6

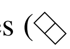
A Let $ABCD$ be a convex quadrilateral, and let $X \in AC \cap BD$ be the intersection of the diagonals. Show that if

$$\text{Area}(AXB) + \text{Area}(CXD) = \text{Area}(BXC) + \text{Area}(DXA)$$

then X is the midpoint of one of the diagonals.

B Consider the following drawing:



where n is the number of peaks at the top. When you draw it, it should have $3n + 1$ cells in total. Find the maximum number of non-overlapping dominoes ( and ) that can fit inside this.

C Let $ABCD$ be a parallelogram. Let P be inside the line segment $\overline{BD}$ such that $PD = 2PB$. Then take the intersections $\{E\} = AP \cap BC$ and $\{Q\} = ED \cap AC$. Show that $AQ = 2CQ$.

D Define a sequence $x_1, x_2, x_3, \ldots$ of positive integers by the following recurrence:

$$x_1 = 2, \qquad x_{n+1} = x_n^2 - 1.$$

Find all pairs (n, k) of positive integers satisfying $x_n = 4^k(4^k + 1)$.

E On a board there are initially written n ones $(1\ 1\ \cdots\ 1)$. A move consists of picking two numbers a, b and replacing them by $a + b, a + b$.

(a) Show that $(a + b)^2 \geq 4ab$ for any real numbers a, b.

(b) Show that after $2n$ moves, there is at least one number greater than or equal to 16.