

Number Sequences

February 20, 2011

For each of the sequence below, find the pattern and continue the sequence with the next few terms:

(a) 2, 5, 8, 11, 14, ____, ____, ____, ...

(b) 0, 3, 8, 15, 24, ____, ____, ____, ...

(c) 1, 2, 4, 8, 16, ____, ____, ____, ...

(d) 2, 6, 12, 20, 30, 42, ____, ____, ____, ...

(e) 3, 7, 13, 21, 31, 43, ____, ____, ____, ...

(f) 4, 8, 9, 12, 16, 18, 20, 24, 25, 27, 28, ____, ____, ____, ...

(g) 4, 11, 30, 67, 128 ____, ____, ____, ...

(h) 1, 2, 6, 24, ____, ____, ____, ...

(i) 1, 1, 2, 3, 5, 8, 13, 21, ____, ____, ____, ...

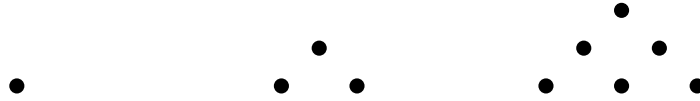
(j) 1, 11, 21, 1211, 111221, 312211, ____, ____, ____, ...

(k) 1, 4, 27, 256, ____, ...

(l) 6, 10, 14, 15, 21, 22, 26, ____, ____, ____, ...

(m) 1, 3, 4, 7, 11, 18, 29, 47, ____, ____, ____, ...

A *triangle number* is any number of stones that can be arranged into a triangle. The first three triangle numbers are 1, 3, and 6, as shown here:



1. Draw arrangements of dots that represent the fourth, fifth, and sixth triangle numbers:

2. Continue the sequence of triangle numbers:

1, 3, 6, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ...

3. Now, below the triangle number sequence, list the sequence you get by subtracting consecutive terms: (For instance, the first two are $2 (= 3 - 1)$ and $3 (= 6 - 3)$.) Draw a picture to explain what's going on.

1, 3, 6, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ...

2, 3, ____, ____, ____, ____, ____, ____, ____, ____, ____, ...

4. Now rewrite the triangle number sequence, and this time below it list the sequence you get by *adding* each pair of neighboring numbers. Draw another picture that explains what's going on here.

1, 3, 6, ____, ____, ____, ____, ____, ____, ____, ____, ____, ...

4, 9, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____, …

5. Now rewrite the triangle number sequence, and this time below it list the sequence you get by adding consecutive terms *starting from the beginning*. For instance the first three terms of the new sequence are 1 ($= 1$), 4 ($= 1 + 3$), and 10 ($= 1 + 3 + 6$). What kind of numbers does this new sequence represent? Draw a picture of the first 3 of these numbers, arranged in a certain shape.

1, 3, 6, _____, _____, _____, _____, _____, _____, _____, _____, _____, …

1, 4, 10, _____, _____, _____, _____, _____, _____, _____, _____, _____, …

Let us call the first triangle number T_1 , the second triangle number T_2 , the third triangle number T_3 , and so forth. So $T_1 = 1$, $T_2 = 3$, $T_3 = 6$, $T_4 = 10$, etc.

6. There are 5 kids, and each pair of kids plays one game of Rock, Paper, Scissors. How many games of Rock, Paper, Scissors are played altogether?
7. The Lake Wobegon High School Marching Band is planning their latest series of marching drills. Alexa says they should arrange themselves in a square, to symbolize a cheese sandwich. Zoe says they should arrange themselves in a triangle, to symbolize pizza.

They argue for a couple minutes, until Melissa interrupts. "Guys, why don't we just do a square and *then* a triangle?"

And that is what they do—every member of the band is used in both the square, and the triangle. How many people are in the Lake Wobegon High School Marching Band, assuming there are less than a thousand people in the band?

8. What's a formula for the n th triangle number? Your formula will have to use " n ", and it's also okay to use "dot-dot-dot" (...) to indicate that a pattern in the formula continues.

9. Can you use your formula to compute T_{30} ?

10. Could you use it to compute T_{1000} ???....

11. Find a formula for T_n that you *can* use to compute T_{1000} (without taking hours!). It might help to look back at the picture you drew to explain problem 4... Remember the number of dots in a square that has n dots on a side, is $n \times n = n^2$.