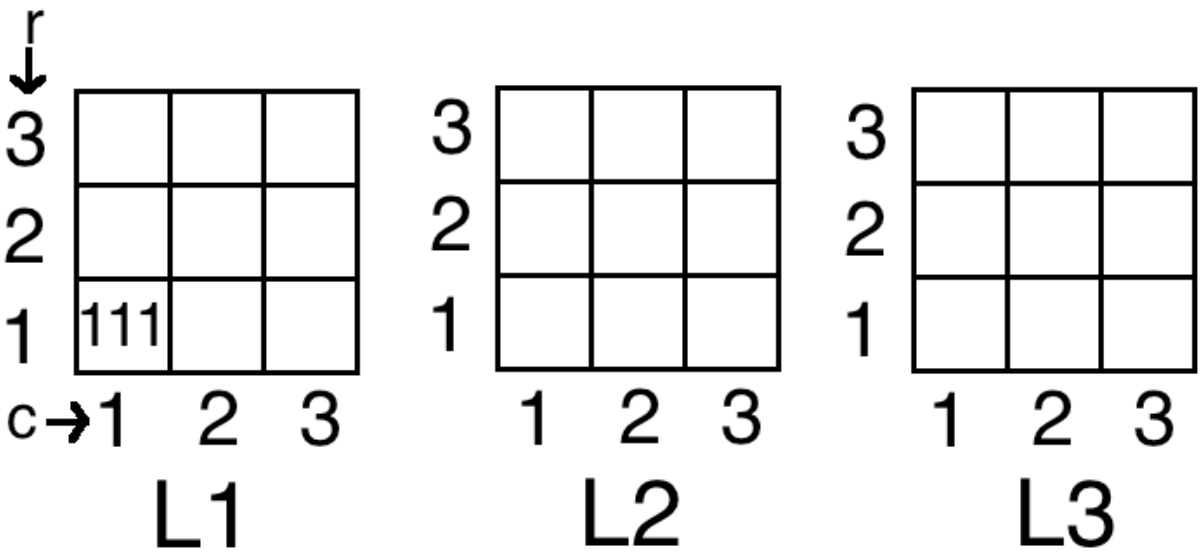
ENCODING SOLIDS USING NUMBERS

JUNIOR CIRCLE 02/13/2011

There are 27 spaces where a cube can be placed when building a solid over a 3x3 base. Let's give a number name to each cube. We can describe the position of a cube using a column (c), a row (r), and a level (l). Here are the c, r, l numbers:



(1) A cube is placed in space 221.
   (a) What is the level?

   (b) What is the column?

   (c) What is the row?

(2) Describe in words where the cubes 222 and cube 113 are.

(3) Write the number names of
   (a) the four corner cubes.

   (b) the center cubes of all six faces in the 3x3x3 cube.

(4) We will say that two cubes have a common face if they are touching each other along a face.
   (a) What cubes have a common face with cube 221?

   (b) What cubes have a common face with cube 222?

(5) What cubes have common vertices
   (a) with cube 321?

   (b) with cube 123?

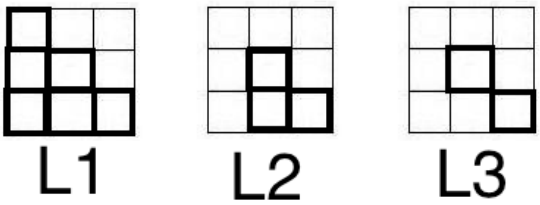
(6) You flip the 3x3x3 cube so that the top and bottom are exchanged.
   (a) What is the new name for cubes
      (i) 133?

      (ii) 211?

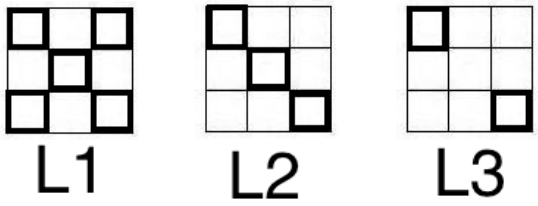
   (b) Are there any cubes whose names do not change when you do this? Explain why.

(7) What cube(s) do not change their name(s) if you put the 3x3x3 cube on its side?

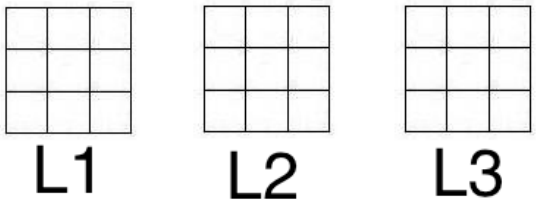
(8) Name all the cubes in the following solid:



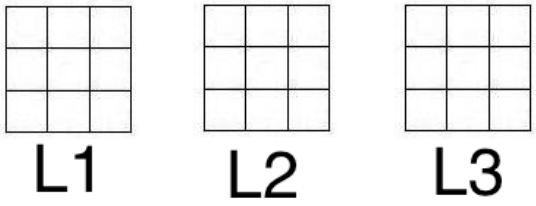
(9) Name all the cubes in the following solid:



(10) Draw the levels of the solid consisting of cubes 111, 211,311, 121, 131, 122, 212, 312, 123, and 313



(11) Draw the levels of the solid consisting of the cubes 111, 221, 311, 131, 231, 222, 232, 112, 223, and 113.



(12) A solid has:

(a) cube 312 in it. What other cube must it have? Why?

(b) cube 213 in it. What other cube(s) must it have? Why?

(13)

(a) A solid contains cubes 112, 122, and 212. What other cubes should be in the solid? Why? What is the smallest number of cubes the solid can have? List all of the cubes in this solid.

(b) A solid contains cubes 133, 313, and 213. What other cubes should be in the solid? Why? What is the smallest number of cubes the solid can have? List all of the cubes in this solid.

(c) A solid contains cubes 113, 132, and 212. What other cubes should be in the solid? Why? What is the smallest number of cubes the solid can have? List all of the cubes in this solid.

(d) A solid contains cubes 112, 113, and 213. What other cubes should be in the solid? Why? What is the smallest number of cubes the solid can have? List all of the cubes in this solid.

(14) Is it possible to build the following solid out of cubes? Explain.
111, 121, 211, 131, 221, 312, 222, 112, 333, 233, and 313.

(15) Given the top projection, find what cubes MUST be in the solid: