Knights and Knaves

Beginners Circle

2-11-18

Knights and knaves are logically minded people. Knights make sure to always tell the truth whenever they say anything, and knaves always lie whenever they say anything. That is, whenever knights speak, they always make precise logical statements that are true. Whenever knaves speak, they make precise logical statements that are false. You're visiting an island inhabited by only knights and knaves, but you don't know who is who! However, with carefully asked questions to everyone you meet, you will be able to figure out who are knights and who are knaves.

Part I: Meeting the islanders.

1. You finally meet someone on the island and immediately ask whether they are a knight. How do they answer?

Yes

2. Realizing your mistake, you think for a little bit. Then you ask, "If I asked if you were a knight, what would you say?", then they reply "no". What are they?

A knave, ~~If~~ Like in problem 1, a knave answers yes when asked directly. So a knave would lie about their answer and say no.

3. Walking along, you finally meet Betsy and Mariam on the island, and begin to have a conversation. Realizing they could be lying about everything, you finally ask about whether they are knights or knaves. In response, Betsy says, "If you asked Mariam if she was a knight, she would say no.", and Mariam says, "If you asked Betsy if she was a knight, she would say yes." What are they?

By ~~part~~ problem 1, ~~Betsy~~ anyone would respond yes, so Mariam is telling the truth. Therefore, Mariam is a knight and Betsy is lying. **Mariam knight. Betsy knave.**

4. Afterwards, you meet Tom and Broderick, and ask them the same question as to Betsy and Mariam. Tom says, "If you asked Broderick if I was a knight, he would say no." Broderick says, "If you asked Tom if I was a knight, he would say yes." What are they?

Tom's statement T ~~T~~ ← should be false if Tom ~~knight~~ knave.

Tkt : Bkv Tkv : Bkn

Tom knight. **Broderick knave**

5. Now you come across another pair of islanders, one named Nan and one named Delly. You ask Nan, "Is Delly a knave?" Rather than respond, Nan whispers the answer to Delly. You ask Delly whether Nan whispered "Yes", and ~~Nan~~ replies, "No!" What is Nan?

Delly 4 Cases:

N:kt D:kt answer "NO!"

N:kt D:kv answer ~~"yes"~~ "NO!"

N:kv D:kt answer "yes"

N:kv D:kv answer "yes"

Nan is a knight

6. A group of Knights and Knaves work for Ye Ole Beer Shoppe. Knights always tell the truth, while Knaves always lie. When each and every one of the employees were asked two questions, they all said the exact same answers:

Q: "How many people work harder than you?" **R:** "At most 10 people work harder than I do."

Q: "How many people get better payment than you?" **R:** "At least 100 people have better salaries."

How many employees worked at Ye Ole Beer Shoppe in total?

~~employees.~~ ~~111~~ 111 employees

The top paid are all knaves, there are 100 of them. The hardest workers are 11 knights all paid the least.

7. 7. I met two men who lived there and asked the taller man "Are both of you Knights?". He replied with a "Yes" or "No", but from his answer, I could not figure out what type of person each man was. I then asked the shorter man "Is the taller man a Knight?". He replied with a "Yes" or "No", and after that I knew which type of person each man was.Were the men Knights or Knaves?

~~Yes~~ Try both knights. They both say Yes. Same with knaves.

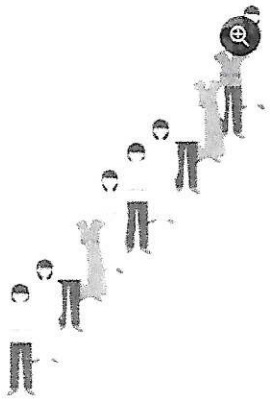
1st kt → No, must be knight. ← no immediately

2nd kn → No, must be knave.

1st kn
2nd kt → Yes, not sure

→ No, only order of "yes" and "no"

Part II: Island Rituals



You come across 11 islanders in a line. While you don't know anything about the first person in line, you know of the remaining islanders **5 of them are knights** and **5 of them are knaves**

You ask the first person "**If I asked if you were a knight, what would you say?**" Instead of responding, they whisper their response to the person behind them.

The second person in line you ask "**Was the thing you were just whispered 'yes'?**" Instead of responding, they whisper their response to the person behind them.

The third person in line you ask "**Was the thing you were just whispered 'yes'?**" Instead of responding, they whisper their response to the person behind them.

This continues until the last person who says "yes"

Is the first person in line a knight or knave?

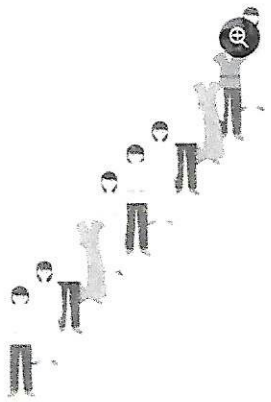
If knight then "Yes", and is reversed 5 times.

Y $\xrightarrow{1}$ N $\xrightarrow{2}$ Y $\xrightarrow{3}$ N $\xrightarrow{4}$ Y $\xrightarrow{5}$ N → WRONG

If knave then "no"

N $\xrightarrow{1}$ Y $\xrightarrow{2}$ N $\xrightarrow{3}$ Y $\xrightarrow{4}$ N $\xrightarrow{5}$ Y → correct!

a knave



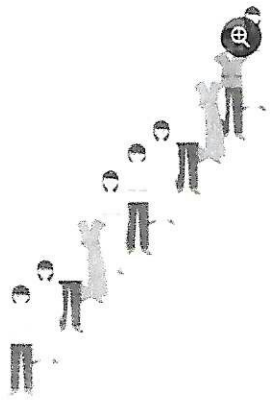
In a group of 10 islanders, each one makes a statement:

- The first says "Exactly 1 of us is a knave."
- The second says, "Exactly 2 of us are knaves."
- The third says, "Exactly 3 of us are knaves."

The pattern continues until the tenth one says, "Exactly 10 of us are knaves."

How many of them are knaves?

9 knaves At most 1 knight, so only true statement is "exactly 9 knaves". There can't be 10 knaves or last person is a knight,



In a group of 10 islanders, each one makes a statement:

- The first says: "**At least** 1 of us is a knave."
- The second says: "**At least** 2 of us are knaves."
- The third says: "**At least** 3 of us are knaves."

The pattern continues until the tenth one says: "**At least** 10 of us are knaves."

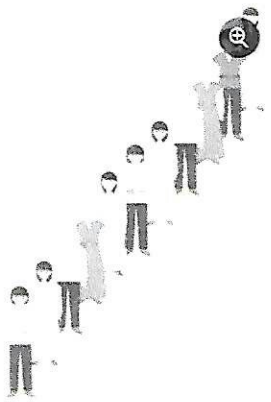
How many of them are knaves?

10th lies, so at least 1 knight

1st is knight, then 2nd → 5th are knights

and 6th → 10th are knaves.

So **5 knaves**



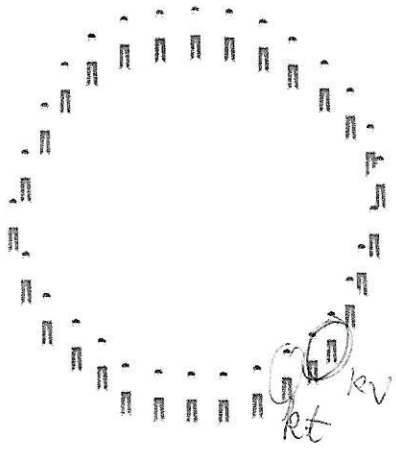
A group of 10 islanders are standing in a line.

- The first five in line state, "Everyone behind me is a knave."
- The last five in line state, "Everyone in front of me is a knave."

How many knaves are there?

If all knaves then paradox.

9 knaves / knight at position 5 or 6.



A group of 30 islanders are standing in a circle.

Each one says "The people to my left and right are both knaves."

What is the **maximum possible** number of knaves?

20, since kt → kv → kt → kt → kv → kv ordering is the max one that produces these statements being said. Replace one kight, and now some knaves are telling the truth

A group of 50 islanders are standing in a line. **You know the first one is a knight.**

- The first one says, "The person behind me is a knight."
- The second one says, "The person behind me is a knave."
- The third one says, "The person behind me is a knight."
- The fourth one says, "The person behind me is a knave."

The pattern continues until the very end of the line, when the fiftieth person doesn't say anything at all. Is that person a knight or a knave?

Tkt Tkv Fkt Fkv

Kt kt kv kv kt kt

1 2 3 4 5 6

Every other pair is the same, so $n = n + 4$

$2 = 2 \cdot 4 \cdot 12 =$ knight = 50