

Homework 3: More graphs and geometry

Claire and Nikita

February 6, 2020

Problem 1.

At a party, there are 15 people. Some of these shook hands and introduced themselves.

Show that there were two people at the party who introduced themselves to *the same number* of people.

Proof. Assume the contrary, namely all the people have different numbers of handshakes. The only possible numbers of handshakes one can make are 0, 1, 2, \dots , 13 and 14. These are 15 numbers, so in order to not have no two people who introduced themselves to the same number of people, all the possible numbers should be present. But it is impossible to have simultaneously one person who shook all the hands and one person who shook no hands. \square

Problem 2.

In a triangle ABC the length of AB is 3.14, the length of $BC = 0.67$. What is the length of AC if we are given that it is integer?

Proof. By triangle inequality, $|AC|$ is between $|AB| - |BC| = 2.47$ and $|AB| + |BC| = 3.81$. The only integer number in this interval is 3, so $|AC| = 3$. \square