Infinity II Homework

Advanced 1

February 4, 2020

Problem 1.
For each of the following functions $f : \mathbb{R} \to \mathbb{R}$, determine if it is injective and/or surjective:

- $f(x) = x$
- $f(x) = -x$
- $f(x) = x - 49$
- $f(x) = x^2$
- $f(x) = x^3$
- $f(x) = sin(x)$

Which ones are bijections?

Problem 2.
For each of the following functions $g : \mathbb{Z} \to \mathbb{Z}$, determine if it is injective and/or surjective:

- $g(n) = n + 5$
- $g(n) = (n - 2)^2$
- $g(n) = 0$
- $g(n) = n \pmod{2020}$

Which ones are bijections?

Problem 3.
Let A be a set with $n \geq 1$ elements. Prove that there are as many subsets of A with even number of elements, as there are with odd number of elements. *Hint:* Construct a bijection.

Problem 4.
Show that $\mathbb{N} \times \mathbb{N}$ is countable. That is, show that there is a bijection between $\mathbb{N}$ and $\mathbb{N} \times \mathbb{N}$. Recall that $\mathbb{N} \times \mathbb{N}$ is a set of all pairs of natural numbers (a, b), $a, b \in \mathbb{N}$. Explain how would you construct such a bijection (even though you don't have to define it formally). A picture might help you, but explain your solution too. *Hint:* Recall the video that we watched. How did we show that $\mathbb{Q}$ is countable? Can you use a similar approach?