Lesson 4: Graphs and Geometry IV

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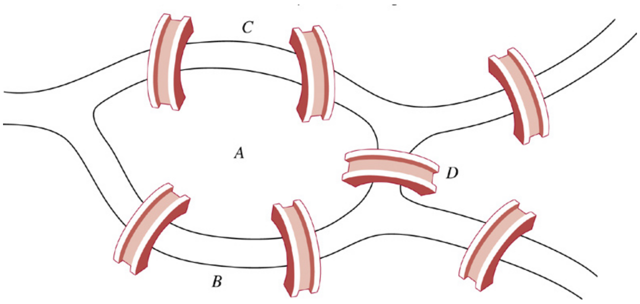
Problem 1.

a) The city of Köningsberg consists of 4 islands and 7 bridges, connecting them as shown on the picture. Is it possible to walk around the city starting and ending at the same island and using every bridge exactly once?



b, c) An Eulerian cycle in a graph is a cycle which passes through every edge exactly once. Show that there exists an Eulerian cycle in a given graph if and only if the graph is connected and all vertices have even degrees. Here the "only if" direction is part b) and the "if" direction is part c).

Problem 2.

Two squares of the chessboard are called connected if and only if it is possible two make a knight's jump from one to the other. Is it possible to walk around the chessboard with a knight while going along every possible connection exactly once? It is not required to start and end at the same spot.

Problem 3.

In a certain country every city has 3 roads connected to it. Show that it is possible to orient every road in such a way that no city has all three roads coming in or out of it.

Problem 4.

A quadrilateral $ABCD$ is called a parallelogram if $AB \parallel CD$ and $AC \parallel BD$. We will show some other defining properties of parallelograms:

a) Show that $ABCD$ is a parallelogram if and only if $AB = CD$ and $AC = BD$.

b) Show that $ABCD$ is a parallelogram if and only if $AB = CD$ and $AB \parallel CD$.

c) Let O be the intersection of the diagonals of $ABCD$. Show that $ABCD$ is a parallelogram if and only if $AO = OC$ and $BO = OD$.

d) Show that the diagonals of a parallelogram $ABCD$ are perpendicular if and only if all of its sides are equal. Such a parallelogram is called a rhombus.

Problem 5.
Let $\triangle ABC$ be isosceles with $AB = AC$ and $\angle BAC = 30°$. Let AD be the median, P be a point on AD and Q be a point on AB such that $PQ = PB$. Find $\angle PQC$.