# Calendar Calculations

Junior Circle 2/11/18

In a normal year, there are 365 days, and in a leap year, there are 366 days. Since leap years occur every 4 years, a year that is divisible by 4 is a leap year (i.e. 2016, 1956, 1472, etc.). However, there are some exceptions to this rule. For centenary years like 2000, 1700, and 1900, the year is only a leap year if it is divisible by 400.

1. Which of the following is NOT a leap year?
   A) 2000
   B) 1584
   C) 1300
   D) 1996

A normal year has 365 days, or 52 weeks plus another day, which we will call an "extra day." Since a leap year has 366 days, it has 52 weeks plus two extra days. As years pass by, these extra days add up. For example, the number of extra days in the year 2000 (leap year) is 2, the number of extra days from 2000 through 2001 (normal year) is 3, the number of extra days from 2000 through 2002 (normal year) is 4, and so on.

Extra days are days that do not make up a full week, so 7 extra days is equivalent to 0 extra days since 7 days are a full week.

2. When is the next year that the calendar will look like the calendar in 2017? (Hint: This is the next year that will start on the same day of the week as 2017.)

To determine the day of the week for any date, we need to calculate the number of extra days and divide by 7 (because there are 7 days in a week). Since January 1, 0001 was a Monday, a remainder of 1 extra day indicates that a given date occurred on a Monday. Thus, the remainder tells us the day of the week for that date according to the following table:

| Number of Extra Days | Day of the Week |
|---|---|
| 0 | Sunday |
| 1 | Monday |
| 2 | Tuesday |
| 3 | Wednesday |
| 4 | Thursday |
| 5 | Friday |
| 6 | Saturday |

There are also numbers of extra days that correspond to each month. A month with 28 days has 0 extra days since 28 is divisible by 7. A month with 29 days has 1 extra day. A month with 30 days has 2 extra days. A month with 31 days has 3 extra days. This pattern is shown in the table below:

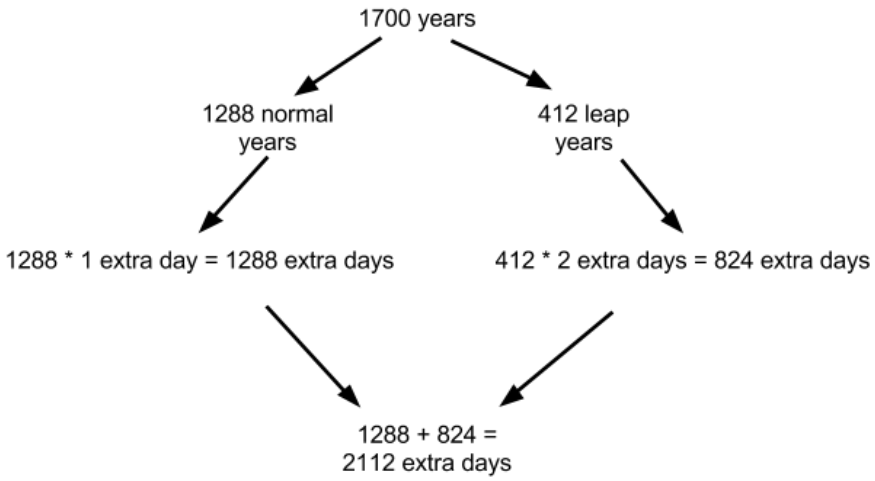
| | Jan | Feb | Mar | Apr | May | June | July | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Normal Year** | 3 | 0 | 3 | 2 | 3 | 2 | 3 | 3 | 2 | 3 | 2 | 3 |
| **Leap Year** | 3 | 1 | 3 | 2 | 3 | 2 | 3 | 3 | 2 | 3 | 2 | 3 |

What day of the week was July 4, 1776?

We will start by computing the number of extra days as follows:

| # of extra days | = | # of extra days in 1700 years | + | # of extra days in **75** full years | + | # of extra days in 6 full months (from Jan-June) | + | 4 days in July |
|---|---|---|---|---|---|---|---|---|

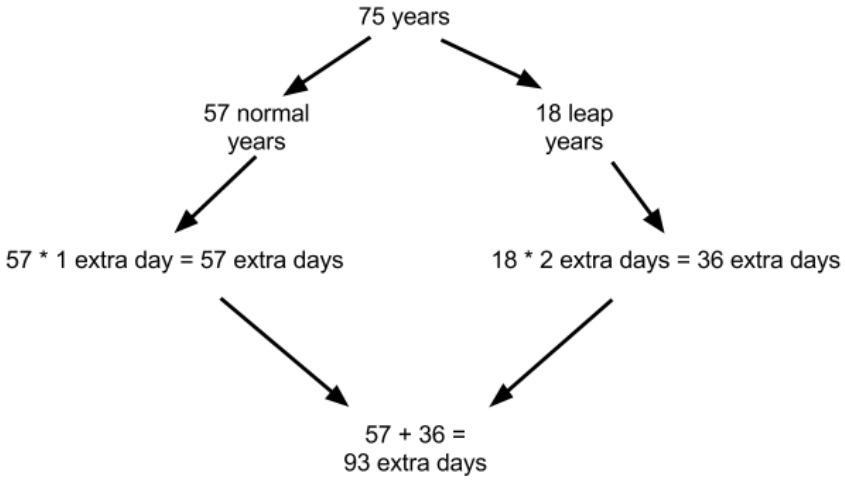
Let's start with the number of extra days in 1700 years. First, $1700 \div 4 = 425$, so there are 425 potential leap years. However, there are 17 centenary years in 1700 years and only 400, 800, 1200, and 1600 are leap years, so out of the 17 centenary years that we counted as potential leap years, $17 - 4 = 13$ of those years are not actually leap years. Therefore, we subtract those 13 years from the 425 potential leap years that we counted earlier, so there are 412 leap years in 1700 years.




$2112 \div 7 = 301\ R5$

This means that there are 301 full weeks of extra days and **5 extra days** in 1700 years.

Next, let's find the number of extra days in 75 years. Since $75 \div 4 = 18\ R3$, there are 18 leap years in 75 years.




Since $93 \div 7 = 13\ R2$, there are 13 full weeks of extra days and **2 extra days** in 75 years.

The next step is to find out how many extra days there are from January through June. Since 1776 is divisible by 4, it is a leap year, so we look at the leap year row of the table on Page 2.

| | Jan | Feb | Mar | Apr | May | June | July | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Normal Year** | 3 | 0 | 3 | 2 | 3 | 2 | 3 | 3 | 2 | 3 | 2 | 3 |
| **Leap Year** | 3 | 1 | 3 | 2 | 3 | 2 | 3 | 3 | 2 | 3 | 2 | 3 |

$3 + 1 + 3 + 2 + 3 + 2 = 14$

$14 \div 7 = 2\ R0$

This means there are 2 full weeks of extra days and **0 extra days** from January through June in the year 1776.

We now have everything we need to know to determine the day of the week of July 4, 1776. We simply add up the 5 extra days in 1700 years, the 2 extra days in 75 years, the 0 extra days from January through June, and the 4 days in July, which gives us 11 extra days.

Since $11 \div 7 = 1\ R4$, 11 extra days is equivalent to 1 full week of extra days plus 4 extra days, so now we just pay attention to the 4 extra days. Looking at the table on Page 1, we see that 4 extra days correspond to Thursday. Thus, July 4, 1776 was a **Thursday**.

3. Instead of dividing by 7 after every step in the example problem, would we still get the same answer if we divided by 7 at the very end? Why or why not?

4. How many extra days are there from year 0 to year 100?

5. How many extra days are there from year 0 to year 400?

6. In a normal year, how many extra days are there from January through April?

7. What day of the week was February 29, 2016?
   a) How many extra days are there in 2000 years? (Hint: Calculate the numbers of regular and leap years.)

   b) How many extra days are there in 15 years?

   c) How many extra days are there in 1 full month (January) in the year 2016? (You shouldn't need to calculate anything.)

   d) How many extra days are there from February 1st through February 29th?

   e) What day of the week was February 29, 2016?

8. What day of the week was December 15, 1938?

a) How many extra days are there in 1900 years? (Hint: Calculate the numbers of regular and leap years.)

b) How many extra days are there in 37 years?

c) How many extra days are there in 11 full months (January-November) in the year 1938? (Hint: Was 1938 a leap year?)

d) How many extra days are there from December 1st through December 15th?

e) What day of the week was December 15, 1938?

**Homework:** Pick any date (i.e. your birthday) and try to figure out what day of the week it was.