PRACTICE TEST 2

LAMC OLYMPIAD GROUP, WEEK 4

Problem 1. Let $\mathscr{C}_1$ and $\mathscr{C}_2$ be two circles intersecting at distinct points P and Q. Let A be on $\mathscr{C}_1$ and D be on $\mathscr{C}_2$, such that the line segment $\overline{AD}$ intersects $\mathscr{C}_2$ again at B, and $\mathscr{C}_1$ again at C. Prove that $\angle APB = \angle DQC$.

Problem 2. **(a)** Suppose that a, b, c, d are integers such that $a + b\sqrt{2} = c + d\sqrt{2}$. Show that we must have $a = c$ and $b = d$.

(b) Consider the expression

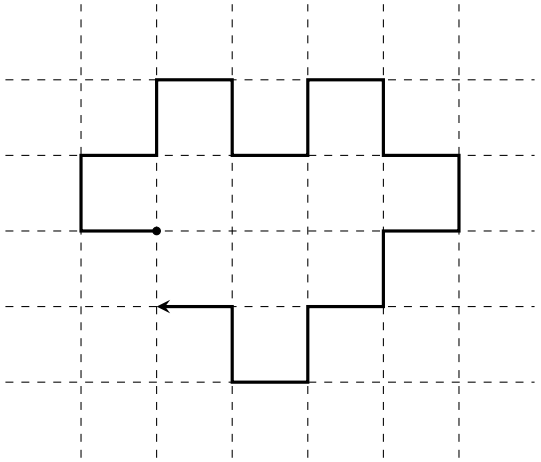
$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n,$$

where $a_0, a_1, \ldots, a_n$ are fixed integers, and x is some variable. Show that if $f(1 + \sqrt{2}) = 0$, then $f(1 - \sqrt{2}) = 0$ as well.

Problem 3. We have an infinite grid, consisting of square cells of dimensions 1×1. A snail starts at some vertex of the grid (i.e. at the intersection in a crosshair $+$) and moves along the lines in the grid, following these two rules:

- Once the snail begins to walk along an edge, it cannot turn around. It must continue until the next vertex ($+$).
- When the snail reaches another vertex, it must change direction by steering either to its left or to its right.

Here is an example of a valid walk:



Show that the snail cannot traverse the same segment in both directions during the same walk (e.g., it cannot move $(2, 3) \to (2, 4)$ and then later also $(2, 4) \to (2, 3)$).