

The Triangle Inequalities:

Let us restate the triangle inequality, and add another inequality for triangles:

- If A, B, C are any points in a plane, then $AC \leq AB + BC$, with equality holding if and only if B lies on the line segment between A and C .
- In triangle ABC , $AB > BC$, then angle C is greater than angle A .

Problems:

1. Give a logical argument based on the above inequalities to establish each of the following:
 - (a) In a triangle, if two sides are equal, then the opposing angles are equal.
 - (b) If AM is a median of triangle ABC and $2 * AM > BC$, then angle BAC is acute.
2. (a) If a, b, c are numbers such that $a + b > c$, $b + c > a$, and $c + a > b$, prove that there exists a triangle whose side lengths are a, b, c .
(b) (*) Prove that if you can form a triangle with side lengths a, b, c , then you can also form a triangle with side lengths $\sqrt{a}, \sqrt{b}, \sqrt{c}$.
(Hint: Which one of the inequalities is “most important”?)
3. The centers of three non-intersecting circles lie on a line. If a fourth circle touches all three given circles at one point each, then its radius is greater than at least one of the radii of the given circles.
4. In triangle ABC , median AM is drawn. Prove that if $AB < AC$, then angle BAM is larger than angle CAM .