Lesson 3: Graphs and Geometry III

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Problem 1.
a) Let ABC be a triangle. Show that if the median and the altitude from A coincide, then ABC is isosceles.

b) Show the same if the angle bisector and the altitude coincide.

c) Show the same if the angle bisector and the median coincide.

Problem 2.
In a quadrilateral $ABCD$ we have $AB = AD$ and $CB = CD$. Show that the diagonals of $ABCD$ are perpendicular.

Problem 3.
a) Let AM be the median of $\triangle ABC$. Show that if $AM = BM = CM$, then $\angle BAC = 90°$.

b) Show the converse: if $\angle BAC = 90°$, then $AM = BM = CM$ where AM is the median.

Problem 4.
Can 9 line segments be drawn on a plane in such a way that each intersects exactly 3 others?

Problem 5.
In a certain country there are 2018 roads going out of every city, in such a way that all cities are connected by the roads network. Show that if any one road is closed for maintenance, all the cities are still connected.

Problem 6.
There are 50 scientists at a conference, and every scientist knows 25 others. Show that it is possible to find 4 scientists and sit them at a round table so that every two neighbors know each other.