Rigid Transformations of the Plane

Some Definitions

Points A, B, C are called *collinear* if they all lie on a single line, and *non-collinear if they don't.*

A *mapping* of the plane is a function which sends points in the plane to points in the plane.

Examples:

A *rigid transformation* or *motion* of the plane is a mapping which preserves the distances between points.

Examples:

The following is a key fact about rigid transformations:

> A rigid transformation of the plane is determined by its action on any triangle (i.e. any three noncollinear points).

Examples:

What examples can you think of? Is that all of them?

Problems

1. (a) Name your two favorite rigid transformations.

   (b) Show the effect of each of your favorite transformations on the figure below:

(The word "BANANA" is part of the figure)

2. Which letters of the English language have axes of symmetry? rotational symmetry?

3. Prove that if a triangle has two axes of symmetry, then it has at least three axes of symmetry.

4. Can a pentagon have exactly two axes of symmetry?

5. It is known that some figure in the plane coincides with itself upon rotation by 48 degrees about a certain point P. Is it necessarily the case that this figure coincides with itself after a rotatioin of 72 degrees around P?

6. If two triangles have the same side lengths a, b, c, prove that one triangle can be moved so that it coincides with the other.

7. Given two circles with equal radius, prove that one can be moved onto the other by means of a rotation. (Which point should you rotate about?)

8. (a) Prove that a translation can be represented as the composition of two reflections in a point.

   (b) What is the composition of two translations? of two rotations? a translation and a rotation?

   (c) Consider the motion which is a reflection in a line ℓ, composed with a translation in a direction parallel to ℓ. Prove that this is not a translation, rotation, or line reflection.