Lesson 2: More graphs and geometry

Konstantin Miagkov

May 14, 2019

Problem 1.
Show that in a $\triangle ABC$ the angle supplementary to $\angle ABC$ equals the sum of angles $\angle BCA$ and $\angle BAC$.

Problem 2.
a) Suppose that in a $\triangle ABC$ we have $AB > BC$. Show that $\angle ACB > \angle BAC$. (Hint: Pick a point D on AB such $BD = BC$, and try to remember something about an isosceles triangle.)

b) Suppose that in a $\triangle ABC$ we have $\angle ACB > \angle BAC$. Show that $AB > BC$.

Problem 3.
In $\triangle ABC$ it is known that $AB = BC$ and $\angle ABC = 108^\circ$. Let D be the foot of the angle bisector of $\angle BAC$. Let E be the intersection of AC and the line through D perpendicular to AD. Show that $BD = BE$.

Problem 4.
Show that the number of states in the US with an odd number of neighboring states is even.

Problem 5.
In a group of 10 people there are 14 pairs who hate each other. Show that it is still possible to assemble a friendly trio of people.