Problem 1.
Is it possible that in a class of 30 people there are 9 people each of whom has 3 friends in the class, 11 people with exactly 4 friends each and 10 people with 5 friends each?

Proof. Let us count the total number of friendships. Let this number be denoted by $e$. Then if we add up the numbers of friends for everybody, we will get $9 \cdot 3 + 11 \cdot 4 + 10 \cdot 5 = 121$. One the other hand, in this number every friendship is counted twice, from the perspectives of both friends. Thus $121 = 2e$. But 121 is odd, so this is impossible. \[ \square \]

Problem 2.
In a group of 5 people each person wrote on the blackboard how many people they are friends with (among these 5 people). The numbers on the blackboard turned out to be 4, 4, 4, 4, 2. Is this possible, or did someone make a mistake?

Proof. Let us number the people 1 through 5. Each of the people 1 through 4 has 4 friends, meaning they are friends with everybody, since there are only 5 people in total. Then they are each in particular friends with person 5. But then person 5 is also friends with everybody, and cannot only have 2 friends. So the answer is impossible. \[ \square \]

Problem 3.
Equal segments $AB$ and $CD$ intersect at a point $O$, so that $AO = OD$. Show that $\triangle ABC = \triangle DCB$.

Proof. If $AO = OD$ and $AB = CD$, then also $BO = CO$. Then the triangle $BOC$ is isosceles, therefore $\angle OBC = \angle OCB$, which is the same as $\angle ABC = \angle DBC$. Using this together with $AB = CD$ and $BC = BC$ we get that $\triangle ABC = \triangle DCB$ by the SAS test. \[ \square \]