Homework 9: Graphs and Geometry

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Problem 1.
Is it possible that in a class of 30 people there are 9 people each of whom has 3 friends in the class, 11 people with exactly 4 friends each and 10 people with 5 friends each?

Proof. Let us count the total number of friendships. Let this number be denoted by e. Then if we add up the numbers of friends for everybody, we will get $9 \cdot 3 + 11 \cdot 4 + 10 \cdot 5 = 121$. One the other hand, in this number every friendship is counted twice, from the perspectives of both friends. Thus $121 = 2e$. But 121 is odd, so this is impossible. $\square$

Problem 2.
In a group of 5 people each person wrote on the blackboard how many people they are friends with (among these 5 people). The numbers on the blackboard turned out to be $4, 4, 4, 4, 2$. Is this possible, or did someone make a mistake?

Proof. Let us number the people 1 through 5. Each of the people 1 through 4 has 4 friends, meaning they are friends with everybody, since there are only 5 people in total. Then they are each in particular friends with person 5. But then person 5 is also friends with everybody, and cannot only have 2 friends. So the answer is impossible. $\square$

Problem 3.
Equal segments AB and CD intersect at a point O, so that $AO = OD$. Show that $\triangle ABC = \triangle DCB$.

Proof. If $AO = OD$ and $AB = CD$, then also $BO = CO$. Then the triangle BOC is isosceles, therefore $\angle OBC = \angle OCB$, which is the same as $\angle ABC = \angle DBC$. Using this together with $AB = CD$ and $BC = BC$ we get that $\triangle ABC = \triangle DBC$ by the SAS test. $\square$