PROBLEM SESSION 2

LAMC OLYMPIAD GROUP, WEEK 3

Problem 1. Set some system of (x, y)-coordinates in the plane. We say that a convex polygon in the plane is *nice* if all of its vertices have integer coordinates. Show that any nice pentagon contains at least one other point of integer coordinates (on its interior or boundary) besides the five vertices.

Problem 2. Let n be a positive integer. Show that

$$1 + x + x^2 + \cdots + x^{2n} > 0,$$

for all real numbers x.

Problem 3. A square $ABCD$ is given. Now suppose that R, S, T, U are points on $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$ respectively, with intersection $X \in \overline{US} \cap \overline{RT}$, such that all four quadrilaterals $ARXU, BSXR, CTXS, DUXT$ admit inscribed circles. Show that X is the center of the square $ABCD$.