Find the cubic!

1. What is the result of expanding the expression $(x+y)^3$?

2. What is the result of expanding the expression $(x-y)^3$?

3. How can you express x^3+y^3 in terms of $x+y$ and xy?

4. How can you express x^3-y^3 in terms of $x-y$ and xy?

5. Find a polynomial with the smallest degree possible which has $\sqrt[3]{2}+\sqrt[3]{4}$ as a root. (Hint: Try cubing it out like in the first problem!)

6. (*) Can you find the other roots of the polynomial you found in the previous problem?

7. Find an equation which as $\sqrt[3]{u}+\sqrt[3]{v}$ as a root.

Find the root!

1. Based on what you saw above, can you see a way to guess a root of the equation $x^3 + px + q = 0$? Try to do what you did in the previous problems, only backwards.

2. Find a root of $x^3 + 6x - 20 = 0$. (This was Cardano's first example!)

3. Find a root of $x^3 - 15x - 4 = 0$.

4. Find the roots of $x^3 + x^2 - 2 = 0$.

5. Find the roots of $x^3 + 4x^2 + 3x = 0$.

6. Find the roots of $2x^3 + 7x^3 - 10x + 3 = 0$.