PRACTICE TEST 1

LAMC OLYMPIAD GROUP, WEEK 2

Problem 1. Let $ABCD$ be a quadrilateral with $\angle DAB = \angle ABC > 90^\circ$, and denote $\{X\} = AC \cap BD$. Let PQ be the parallel line to CD passing through X, where $P \in AD$ and $Q \in BC$. Show that

$$\frac{PX}{QX} = \frac{AD}{BC}.$$

Problem 2. Show that for positive integers $m \geq n$, the binomial coefficient

$$\binom{2^m - 1}{2^n - 1} = \frac{(2^m - 1)!}{(2^n - 1)!\,(2^m - 2^n)!}$$

is odd.

Problem 3. Suppose that n is a positive integer whose digits are all either 0 or 4. Show that one cannot write $n = a^4 + b^4 + c^4$ for any integers a, b, c.