# Hints for olympiad-style problems, week 2

**Problem 1.** Try to draw the region of the square $S$ that corresponds to the event $1/5 \leq d(P) \leq 1/3$.

**Problem 2.** First figure out which $y$ satisfy $f(y) = 6$.

**Problem 3.** First write down what it means for the number with the leftmost digit deleted to be $1/29$ of the original number. Now notice that $10^n$ is never divisible by 7 for any $n$.

**Problem 4.** Use condition (3) to conclude that $f(x, y) = \frac{y}{y-x} f(x, y - x)$ whenever $y > x$.

**Problem 5.** Triangles BCA and DCM are similar.

**Problem 6.** Let $a_n$ be the number of acceptable strings of length $n$ ending in A and $b_n$ be the number of acceptable strings of length $n$ ending in B. Try to get recursive formulas for $a_n$ and $b_n$.

**Problem 7.** If three numbers $a, b, c$ have the property that none of them is more than twice as big as any other, then they form a triangle. Now use pigeonhole principle.

**Problem 8.** If you replace $x$ with $90 - x$, the set of three numbers in question doesn't change, so you can assume $0 \leq x \leq 45$. Now what is the condition for forming a triangle?

**Problem 9.** $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$. Use the binomial theorem.

**Problem 10.** Apply Problem 9 with $2n+1$ in place of $n$ and and consider the resulting expression as a polynomial in the variable $\theta$.

**Problem 11.** Deduce that $\csc^2\theta > 1/\theta^2 > \cot^2\theta$ and apply Problem 10 with suitably chosen values of $\theta$.

**Problem 12.** First prove the formula $S_n = 2^{n-1}(n + 1)$.

**Problem 13.** The points $M, P, Q, C$ all lie on a common circle.

**Problem 14.** Use the formula that if $B = (B_{i,j})$ is any $d \times d$ matrix, then

$$\det(B) = \sum_{\sigma \in \mathrm{Perm}(d)} \left( \prod_{i=1}^{d} B_{i,\sigma(i)} \right)$$

where $\mathrm{Perm}(d)$ is the set of all permutations of the set $\{1, 2, \ldots, d\}$.