Problem 1.
Suppose $a$ has quotient $q$ and remainder $r$ when divided by $b$. What is the quotient and remainder of $3a$ when divided by $3b$?

Problem 2.
a) Use the Euclidean algorithm to find the gcd of the following pairs of numbers: $(52, 47), (124, 1024), (201, 315)$
b) Find at least one pair of integer solutions for each of the following equations
   \[
   \begin{align*}
   52x + 47y &= 1 \\
   124x + 1024y &= 4 \\
   201x + 315y &= 3
   \end{align*}
   \]
c) Given two positive integers $a, b$, describe how to find at least one solution to the equation $ax + by = \gcd(a, b)$.

Problem 3.
In this problem, you can assume the conclusion of problem 2c): For any two positive integers $a, b$ there exists an integer solution $x, y$ to the equation $ax + by = \gcd(a, b)$.
a) Let $a$ be an integer and $p$ be a prime number that does not divide $a$. What is $\gcd(a, p)$?
b) \textit{(Euclid’s lemma)} Suppose $a, b$ are positive integers and $p$ is prime such that $p \mid ab$. Prove that $p \mid a$ or $p \mid b$. (Hint: assume that $p$ does not divide $a$. Then by part a) you know $\gcd(a, p)$. Use that and 2c)

Problem 4.
Using problem 3b), it is possible to show that any positive integer has a unique prime factorization: it can be written as a product of primes in a unique way. You can use this fact in this problem.
a) Find the smallest integer greater than 1 that has remainder 1 when divided by 2, 3, 5, 7.
b) Find all such positive integers.