1. Simplify to a power of 3: $3^3 + 3^3 + 3^3 =$ $3 \cdot 3^3 = 3^4$

2. Simplify to a power of 15: $3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$ $(3 \cdot 5)^3 = 15^3$

3. Solve to a number: $2^5 \cdot 5^2$ $(2 \cdot 5)^2 \cdot 2^3 = 800$

4. Solve to a number: $2^2 + 2^1 + 2^0$ $4 + 2 + 1 = 7$

5. Simplify to a power of 10: 1,000,000,000 10^9

Pattern

6. What ones digit do you get when you multiply out 123^{125}? ←

123^1 3 $123^5 = 3$
123^2 9 $123^6 = 9$
123^3 7 $123^7 = 7$
123^4 1 $123^8 = 1$

so: $123^{125} \Rightarrow 3$

7. Simplify to a power of 2: $2^5 + 4^3 + 5^2 + 6^1 + 7^0$

$2^5 + 2^6 + 25 + 6 + 1 = 2^5 + 2^6 + 2^5 = 2^7$

8. Each of the numbers 2, 3, 4 and 5 are randomly assigned to a, b, c and d. What is the largest possible value of:

Test them.

$a^b + c^d$

$4^5 + 3^2 = 1033$

Is this surprising?
$4^5 > 5^4$, but $3^2 > 2^3$

9. What is the last digit (the ones digit) for this sum, if you write it out and add it all up?

$7^1 = 7$
$7^2 = 9$
$7^3 = 3$
$7^4 = 1$

Adds to zero, and repeats!

$7^1 + 7^2 + 7^3 + \dots + 7^{2010}$

$7^1 + \dots + 7^{2008} \Rightarrow 0$ so $7 + 9 \Rightarrow 6$

10. Solve this, and then tell me the sum of the digits of your answer. For instance, if your answer is 19, the sum of the digits would be 20; and if you answer is 145, the sum is 10.

$10^1 - 1 = 9$
$10^2 - 1 = 99$
$10^3 - 1 = 999$

$10^{2010} - 1$

so $10^n - 1 = \overbrace{9999\dots9}^{n}$

Thus $2010 \times 9 = 18{,}090$

11. What is the remainder when 3^{777} is divided by 7?

$3^1 \div 7 \Rightarrow 3$
$3^2 \div 7 \Rightarrow 2$
$3^3 \div 7 \Rightarrow 6$
$3^4 \div 7 \Rightarrow 4$

$3^5 \div 7 \Rightarrow 5$
$3^6 \div 7 \Rightarrow 1$ ← multiples of 6!
$3^7 \div 7 \Rightarrow 3$

so $3^{777} \Rightarrow 6$

12. The integer 64 is both some number squared (8) and some number cubed (4). Find the next integer that has this same property.

$2^6 = 2^3 \cdot 2^3$ and $2^2 \cdot 2^2 \cdot 2^3$

Next number will be 3^6!

$3^3 \cdot 3^3$ or $3^2 \cdot 3^2 \cdot 3^2$

729

13. How many 30s must be added together to get a sum equal to 30^3?

$30 \cdot \underbrace{30 \cdot 30} = 900 \cdot 30$

900

14. What is the remainder when 2^{100} is divided by 10?

$2^1 \Rightarrow 2$

$2^2 \Rightarrow 4$

$2^3 \Rightarrow 8$

$2^4 \Rightarrow 6$ ← Multiples of 4

$2^5 \Rightarrow 2$

$2^6 \Rightarrow 4$

$\vdots$

6

Notes:

- #6, #11, and #14 all have repeating patterns. Find the pattern and you have the answer.

- #9 also has a pattern: every four numbers add to a ZERO in the 1's place.

- #7 manipulates to

$$= 2^5 + 2^6 + 32$$

$$= 2^5 + 2^6 + 2^5$$

$$= 2 \cdot 2^5 + 2^6$$

$$= 2^6 + 2^6$$

$$= 2^7$$