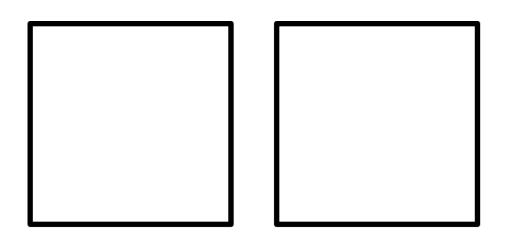
PROBLEM SOLVING

JUNIOR CIRCLE 01/09/2011

(1) Given two equal squares, cut each of them into two parts so that you can make a bigger square out of four parts that you got by cutting the two smaller squares.

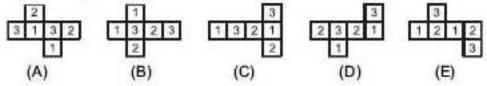


(2) Jack gets paid \$48 for each day that he works. He has to pay \$12 for each day that he misses work. After 30 days, Jack was told that he does not owe anything and will not be paid anything. How many days did he work?

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(3) Cross out one digit in the number 1829 so that you get the smallest possible number.

(4) Alyssa wrote a number on each of the sides of a cube so that the same number is written on the opposite sides of the cube. Which of the following pictures can you get if you this cube along some of the edges and unwrap it?



(5) For a field trip each student packed a snack consisting of either an apple, or crackers, or apple and crackers. There were a total of 57 apples and 36 bags of crackers. 12 students packed both an apple and a bag of crackers. How many students went on the field trip?

(6) There are 40 students in the two groups of the Junior circle. 30 students can swim. 27 students can play chess. 5 students can't swim and don't play chess. How many students can both swim and play chess?

- (7)
- (a) There are 20 students sitting around 4 tables in a math circle. Is it true that there is a table where 5 or more students are sitting?
- (b) There are a total of 114 students in 5 classrooms in an elementary school. Is it true that there is a classroom with over 20 students in this school?

(8) A traveler has to cross the desert along a 80 miles route. Each day he can cover 20 miles. Since there is no wated in the desert, he has to leave water and food for himself. He can carry water supply which is enough for 3 days. How many days does he need to cross the desert? (Obviously, he has to come back as he can not carry the water supply enough for the entire trip). Hint: Try drawing a picture of the road!

- (9) Try these problems by drawing a picture for part (a) first and use what you notice.
 - (a) There are 7 televisions sharing cable. Can you connect them to each other in such a way that each is connected to exactly 4 other televisions?

(b) There are 15 computers in a computer class. Can you connect them to each other in such a way that each is connected to exactly 5 other computers?

- (10) Jane painted a cube with 3 cm sides red. She then sawed it into cubes with 1 cm sides.
 - How many cubes did she get in all?
 - How many cubes have exactly three sides painted?
 - How many cubes have exactly two sides painted?
 - How many cubes have exactly one side painted?
 - Are there any cubes that are not painted at all?
- (11) Shannon wrote 20 on a piece of paper. There are 21 students in the math circle. The piece of paper is being passed around the classroom so that each student changes it exactly once by adding or subtracting 1. Can they get 30 at the end?

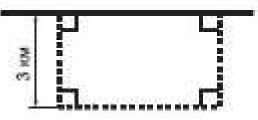
(12) You are on an island and there are three crates of fruit that have washed up in front of you. One crate contains only apples. One crate contains only oranges. The other crate contains both apples and oranges. Each crate is labeled. One reads "apples", one reads "oranges", and one reads "apples and oranges". You know that NONE of the crates have been labeled correctly - they are all wrong. How many fruits do you need to take out to label the crates correctly? Explain how you would do it.

(13) Can you find how old Kate and her son are if:

- 5 years ago Kate was 5 times as old as her Son;
- 5 years from now Kate's age will be 8 years less than three times the corresponding age of her Son.

(14) Using 8 exactly eight times make a 1000. You can use any mathematical symbols ($+, -, \div, \star$).

(15) Due to road construction, the street that Daniel walks everyday is closed. He now must take the detour and travel along the dotted edges in the picture. By how much does this increase the length of the walk?



(16) The sum of two numbers is 3 more than one of the numbers and 4 more than the other number. What is the sum of the two numbers?

(17) You know that:

- When it rains, the cat is in the house or in the attic.
- When the cat is in the house, the mouse is in its hole and the cheese is in the refrigerator.
- If the cheese is on the table and the cat is in the attic, the mouse is in the house.
- Right now it is raining and the cheese is on the table. What can you conclude?
- (a) the cat is in the house
- (b) the mouse is in the hole
- (c) the cat is in the house and the mouse is in the hole
- (d) the cat is in the attic and the mouse is in the house

(18) You need to boil eggs for exactly 9 minutes, or else the visiting Duchess will complain, and you will lose your job as head chef. But you have only 2 Hourglasses, one measures 7-minutes, and the other measures 4-minutes. How can you correctly measure 9 minutes?

(19) You have a balance scale with four weights. With these four weights you must balance any whole number load (in ounces) from 1 all the way up to 40. How much should each of the four weights weigh? (You may place weights on both sides of the scale at the same time.)