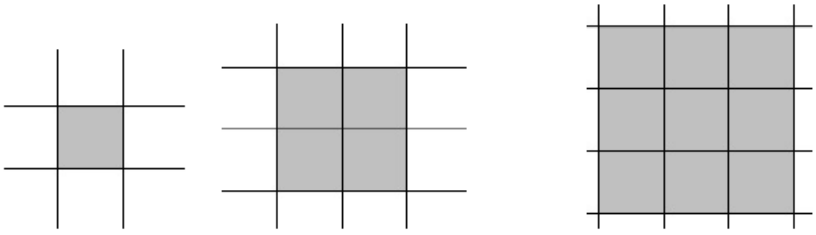
Similar Shapes and Gnomons

January 28, 2018

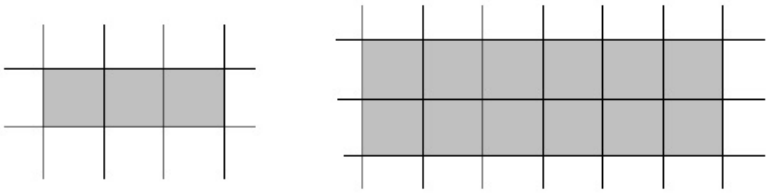
1. Similar Shapes

For now, we will say two shapes are *similar* if one shape is a "magnified" version of another. All the dimensions must be magnified by the same factor. For example, this means that if the length is magnified by a factor of 3 then the width is also magnified by a factor of 3.

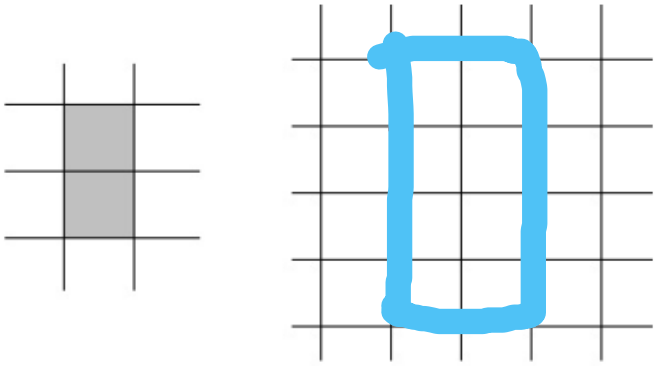
1. In the picture below, the square on the left is of size 1×1. The square in the middle is *similar* to the first square. The square on the right is also *similar* to the first square with a magnification factor of 3. All three squares are *similar*.



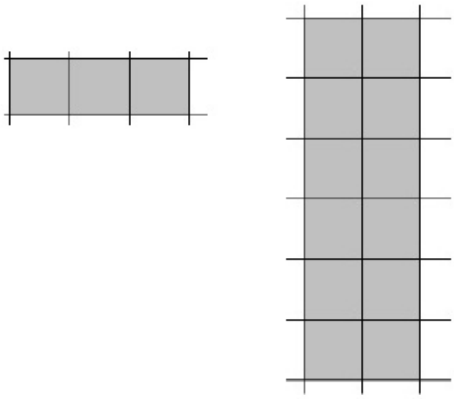
2. Below, the rectangle on the left is of size 3×1. The rectangle to the right is *similar* to the one on the left: both the width and the length of the rectangle are twice those of the original rectangle on the left.



3. Below is a picture of a rectangle of size 1×2. Draw and shade in a rectangle on the right that is *similar* to the original rectangle (magnified twice).



(a) Do you think that the two rectangles below are similar? Why or why not?

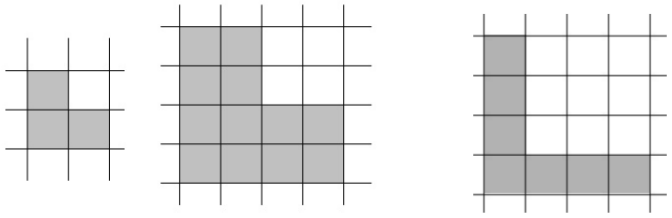


yes, the two rectangles are similar because the second rectangle's width and length are double the first one. (magnified by factor of 2)

(b) How are they different compared to the pair above?

The first pair are in the same direction, whereas the second pair is rotated 90 degrees. They are both magnified by a factor of 2, though the original dimensions differ.

4. We can also find *similar* shapes that are not just squares or rectangles. Consider the three L-shapes below :



We'll call the three shapes A, B, and C (in that order).

   (a) Can you say that the "L" shapes have the same shape?

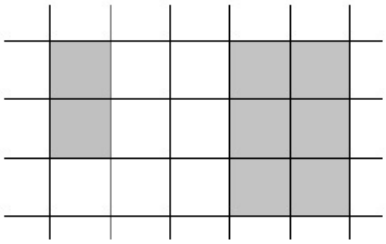
No, only the first two are similar, where the middle one is magnified by 2 of the first one. However, the right is only extending the legs of the "L"

5. When trying to decide if two shapes are *similar* by our definition, think about the following questions:

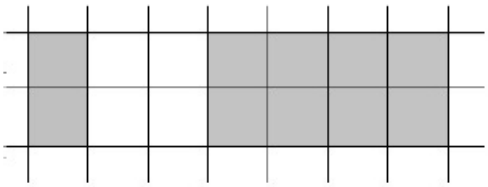
   (a) Does the larger shape look like a magnified version of the smaller?

   (b) If the width of the shape is magnified by a certain factor, is the length also magnified by that same factor?

   Are either of the following pairs of rectangles similar?

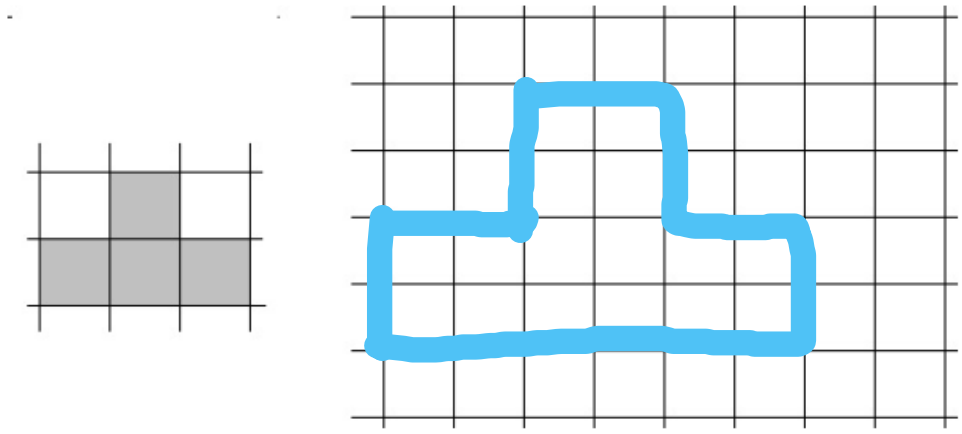


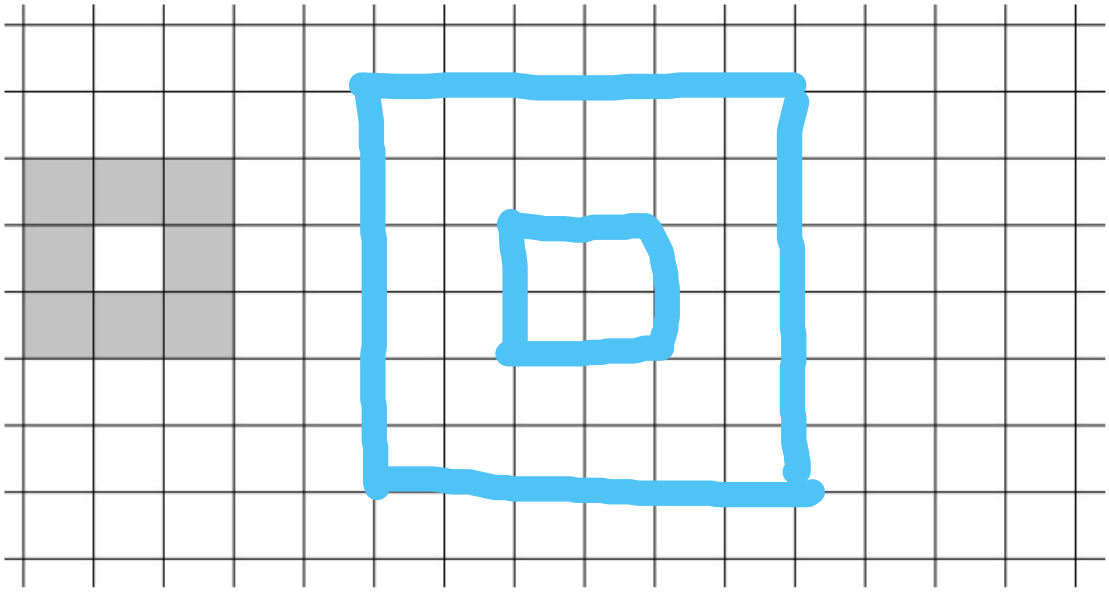
This pair is not similar because the width is magnified by 2 but the length is magnified by 1.5



This pair are similar because all dimensions are magnified by 2

6. Use the grids on the right to draw a *similar* shape that looks like the original, magnified twice.



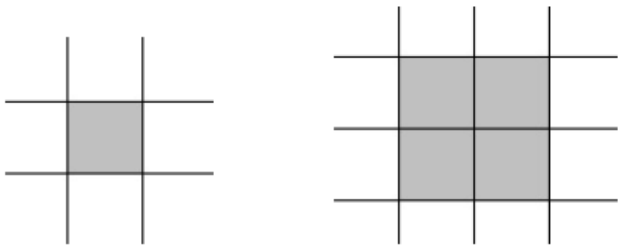


2. Gnomons

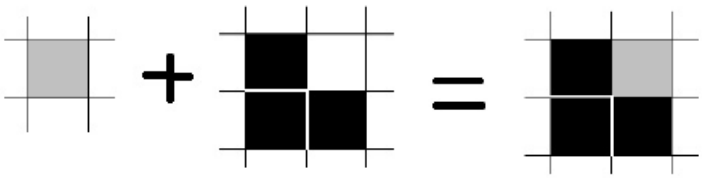
Suppose we have a shape we want to magnify. What is a "gnomon?"

A gnomon is a piece that you can attach to the original shape to create a shape that is *similar* to the original. (i.e., a magnified version of the original)

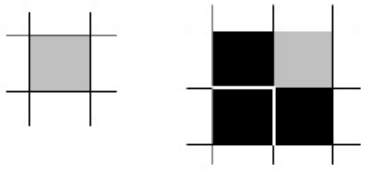
1. Consider two similar squares from the first page:

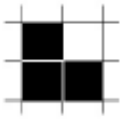


What piece could we attach to the square on the left to make the square on the right? Below is one possibility:

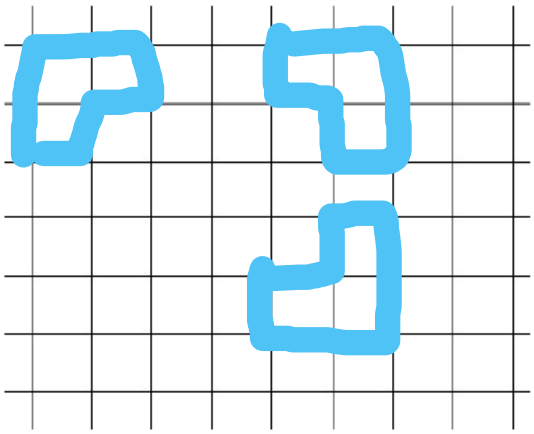


When the dark-colored L-shape is attached to the original 1×1 square, we get a new square which is twice as big.

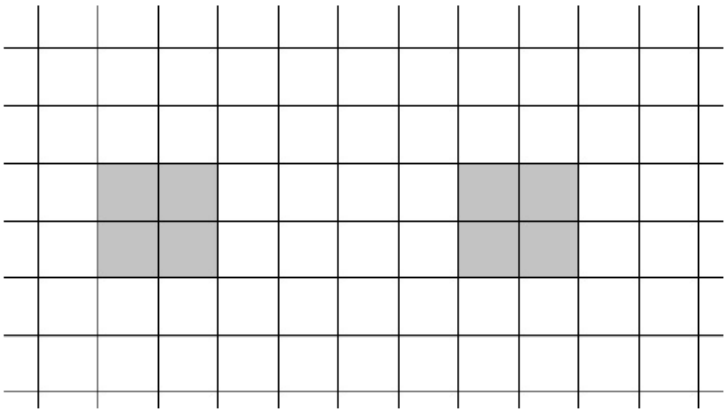


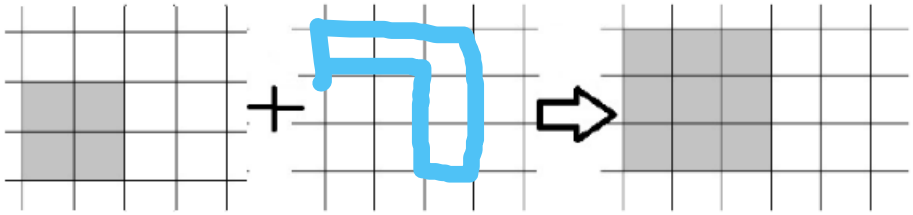
That L-shape  is therefore known as a *gnomon* for the square.

(a) Find 3 other *gnomons* that make a 1×1 square into a 2×2 square:

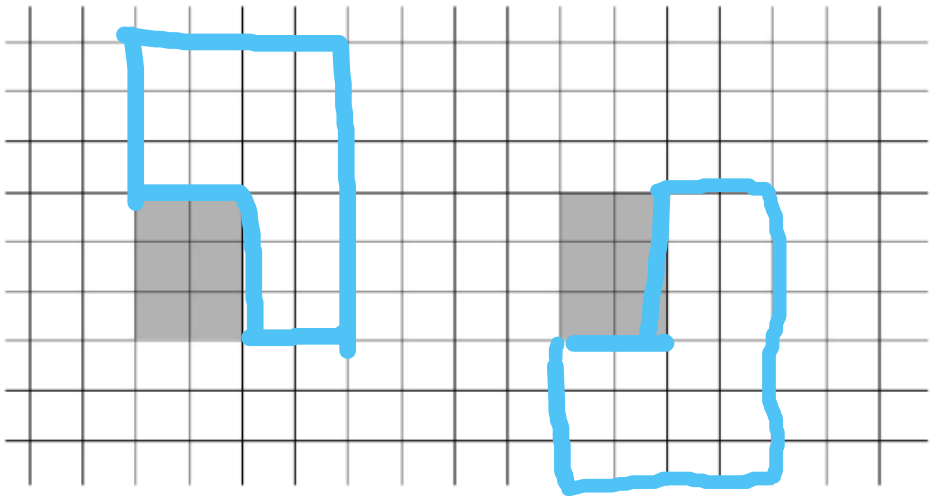


(b) We learned above that a 2×2 square and 3×3 square are also similar. Find the piece (gnomon) that can be added to the 2×2 square to make it a 3×3 square:

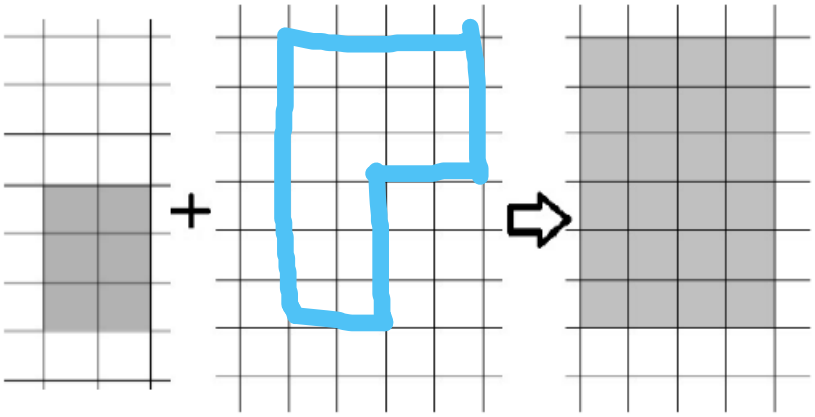




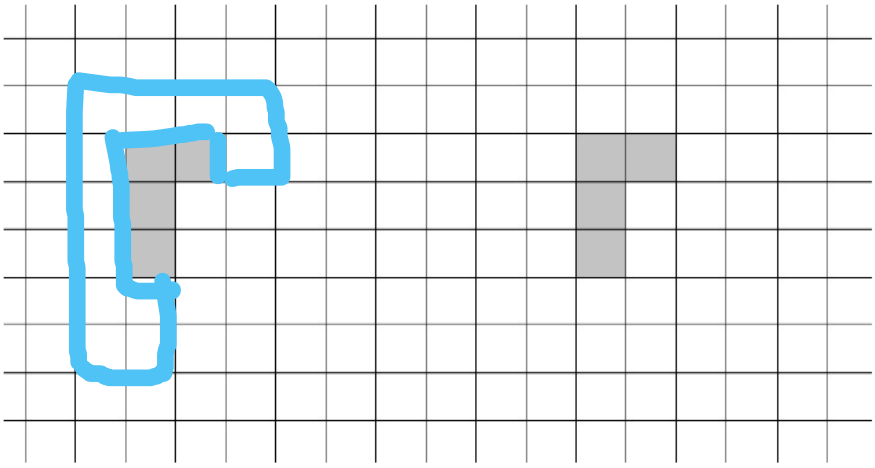
2. Find a gnomon for this rectangle that magnifies it by a factor of two (the resulting *similar* rectangle should be twice as big as the original). You can use the grids below.



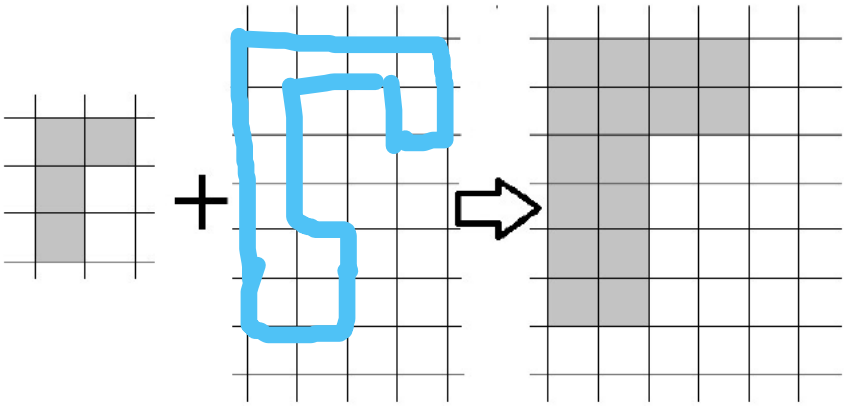
(a) So what does the gnomon look like? Draw it on the middle grid below:



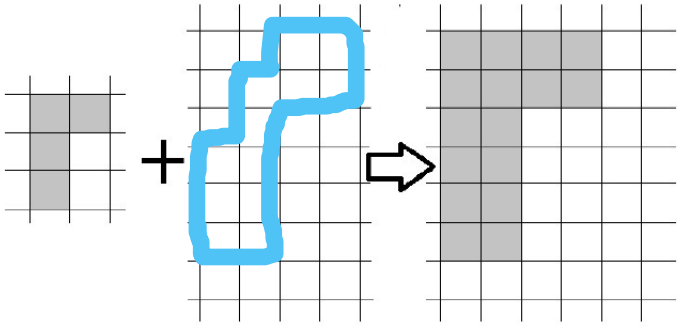
3. Find a gnomon that, when attached to the shape below, creates a *similar* shape that is the original magnified by a factor of 2.



(a) What does the gnomon look like? Fill in the blank spot below

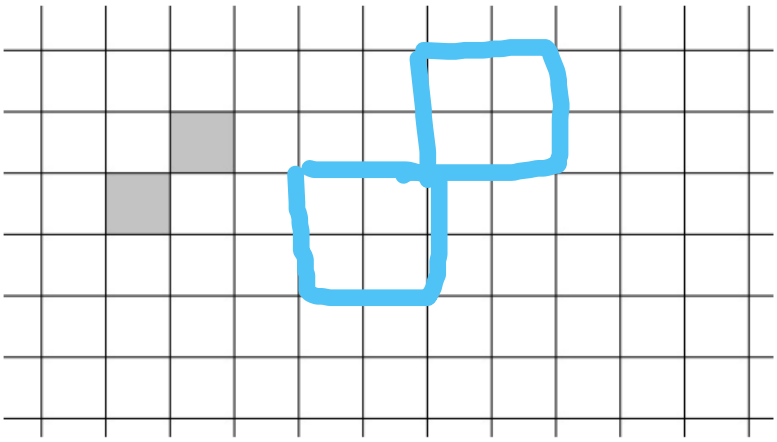


(b) Draw a different gnomon. (Hint: start by tracing the original in the new shape.)



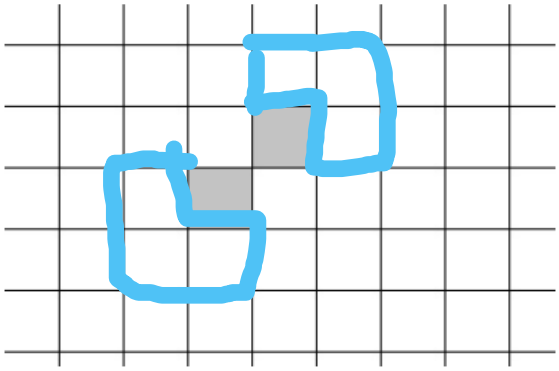
3. Disconnected Gnomons

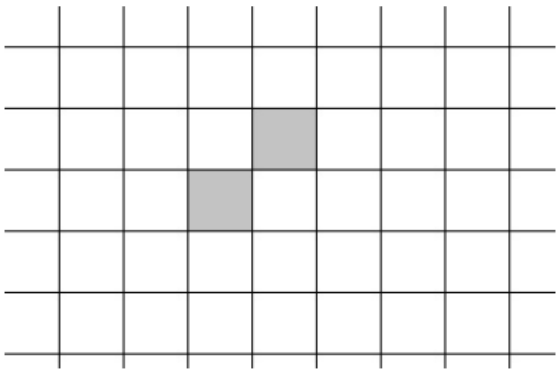
1. Magnify the shape below by a factor of two:



(a) Looking at the two shapes above, can you find one piece that will produce the bigger shape when attached to the original? No

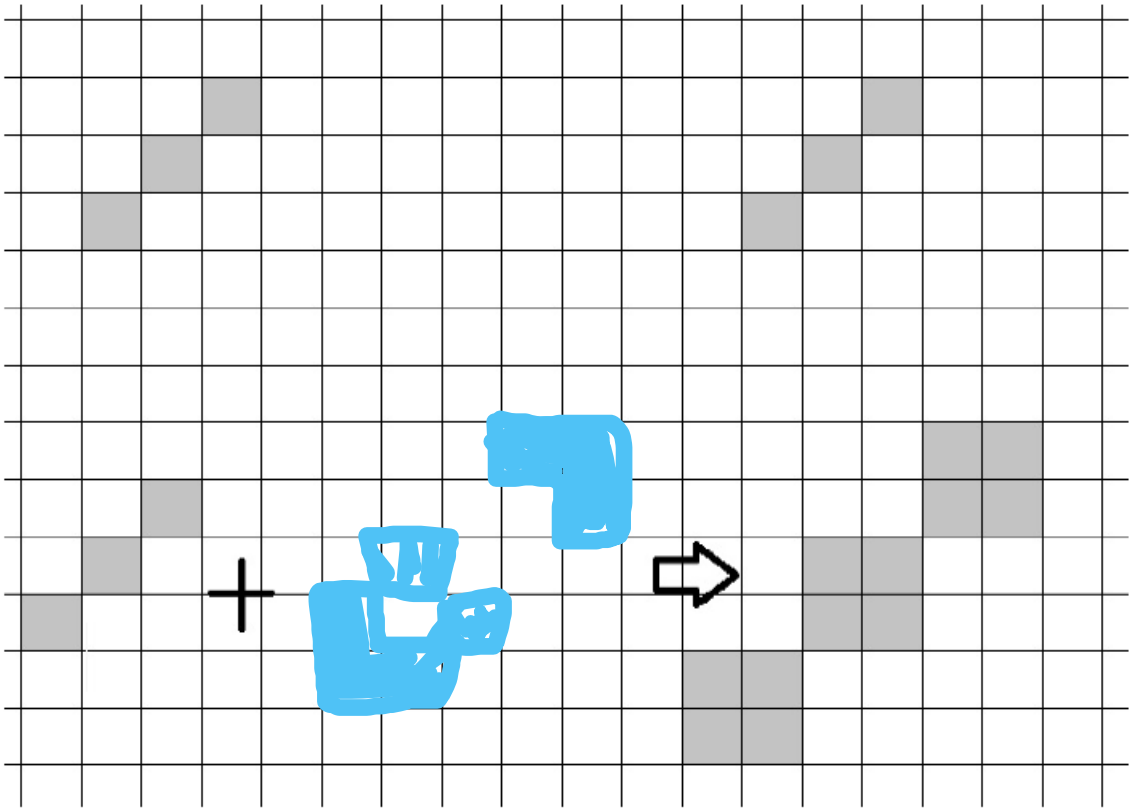
(b) What if you are allowed to add two pieces? Draw them in below. (The two added pieces may be different) yes



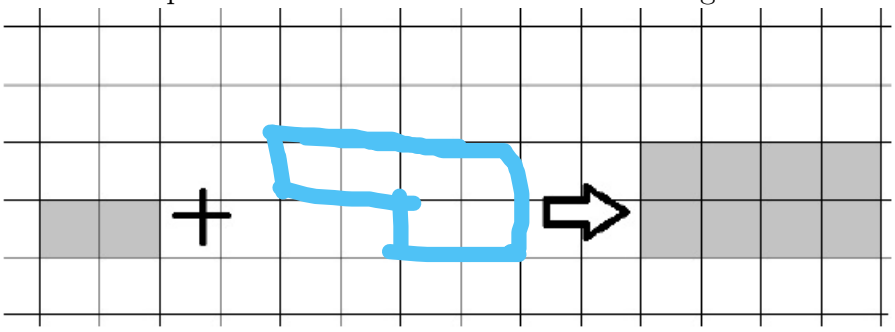


2. As it turns out, some shapes do not have a connected gnomon (a gnomon which consists of a single piece). Even though there is no gnomon which is a single piece. we can find 2 or more pieces that can be added to make a magnified version of the original

   (a) Does this shape have a connected or a disconnected gnomon? disconnected

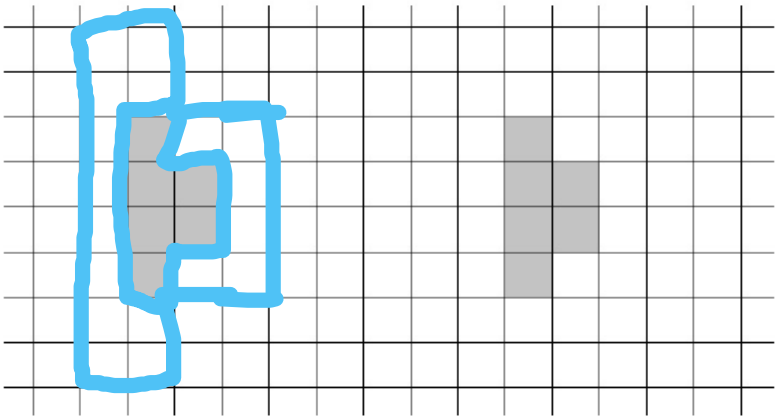


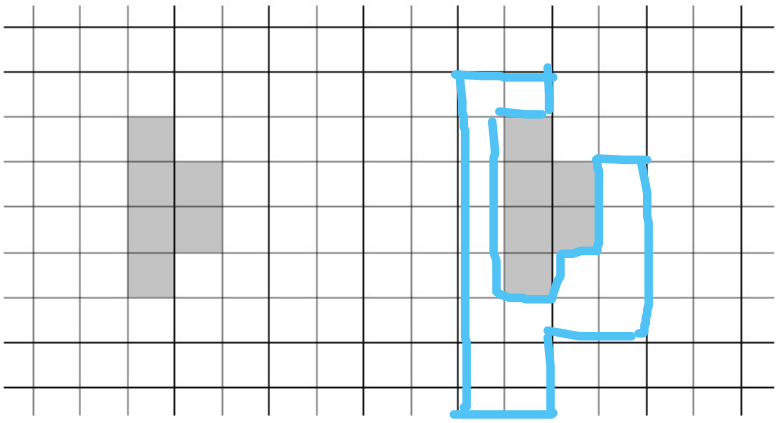
   (b) Does this shape have a connected or a disconnected gnomon?

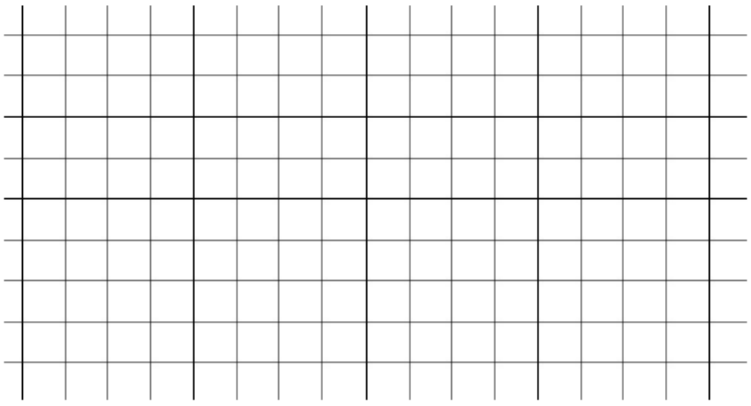


connected

3. Sometimes, a shape can have both a connected and a disconnected gnomon. For the shape below, create a similar image on the right with a magnification factor of 2. Can you find one connected gnomon and one disconnected gnomon for this shape?

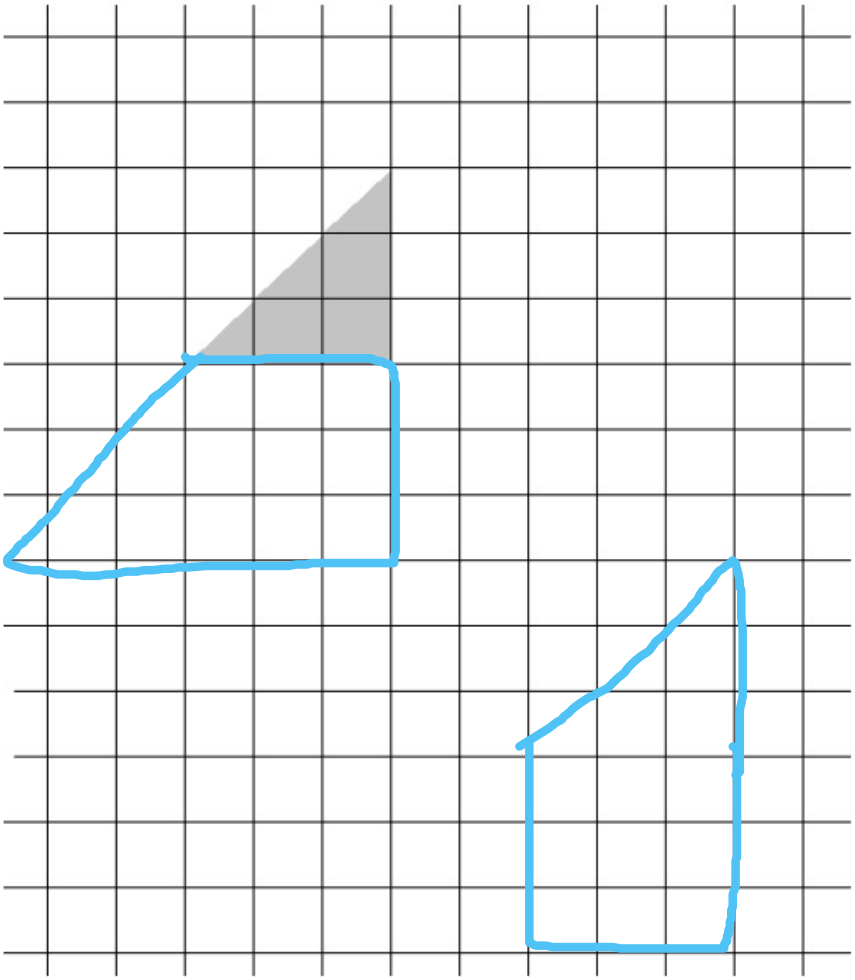






4. Challenge

1. Explain why all squares are similar to eachother. Can you think of any other shape that has this property? All sides increase in length/width by the same magnitude! Another example would be a rhombus!
2. Find two gnomons for the following triangle, which has two equal sides. (If it seems confusing, remember that this triangle is exactly one half of a square cut diagnally.)



5. Homework

Come next week with a shape (not a square or rectangle) with a connected gnomon that, when connected to the original shape, *triples* the size of the shape. Draw the original shape, gnomon, and the larger *similar* shape separately. It may help to use graph paper.