Lesson 7: Remainders, Primes and the Euclidean Algorithm

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May 5, 2019

Problem 1.
Compute: the remainder of -7 when divided by -2, the remainder of -153 when divided by 15, the remainder of 153 when divided by -15.

Problem 2.
Show that a prime number greater than 3 can be expressed as $6n + 1$ or $6n + 5$ for some nonnegative integer n.

Problem 3.
a) Find 3 distinct positive integers greater than 1 such that product of any two is divisible by the third.

b) Show how to construct infinitely many such examples.

Problem 4.
a) Let a, b be positive integers such that $a \leq 100$ and $b \leq 100$. Show that computing $\gcd(a, b)$ with the Euclidean algorithm takes at most 20 steps.

b) Show that in fact it takes at most 11 steps.