Powers and Exponents (Part Two)

November 2019

1. Remember, exponents are just a shorthand.

1a. $4^5 =$ ___ $\cdot$ ____ $\cdot$ ___ $\cdot$ ___ $\cdot$ ___

1b. $4^5 \cdot 4^2 =$

1c. $5^3 \div 5^1 =$

1d. $(3^2)^3 =$

1e. $(4^{10})^3 =$

2. We can compare numbers easily if they have the same BASE.

2a. Which is bigger: 2^5 2^9 2^3

2b. Which is bigger: 13^5 13^9 13^3

2c. Which is bigger: $13^3 \cdot 13^2$ $13^6 \cdot 13^3$

3. We can compare numbers if we can cause them to have the same BASE.

3a. Which is bigger: 8^2 2^4

3b. Which is bigger: 4^3 16^9

3c. Which is bigger: 81^4 3^5

3d. Which is bigger: $125^1 \cdot 5^3$ $5^2 \cdot 5^5$

4. We can compare numbers easily if they have the same EXPONENT.

4a. Which is bigger: 5^{15} 6^{15}

4b. Which is bigger: 8^{16} 9^{16}

4c. Which is bigger: $3^5 \cdot 5^5$ $4^5 \cdot 8^5$

4d. Which is bigger: $9^6 \cdot 2^6$ $3^6 \cdot 7^6$

5. We can compare numbers if we can cause them to have the same exponent.

5a. Which is bigger: 8^3 5^9

5b. Which is bigger: 16^2 3^8

6. Sometimes we can easily compare numbers even if the bases and exponents are different.

6a. Which is bigger: 8^3 7^2

6b. Which is bigger: 9^{14} 6^{11}

7. When numbers are multiplied together, sometimes we can compare them most easily be re-writing them down using "factors" and exponents.

7a. Which is bigger: 9×81 18×12

7b. Which is bigger: 25×36 81×20

8. And sometimes there are cool puzzles to think through.

8a. Without doing the math, can you predict which of these is ***greater***?

$(7^7 + 8^8)$ 9^9

8b. Without doing the math, can you predict which of these is ***greater***?

2^{333} 3^{222}

8c. The number 100 has 2 zeros on the righthand side of the number. The number 2,800 has 2 zeros on the righthand side of the number. The number 13,400,000 has 5 zeros on the righthand side of the number. Without doing the math, can you predict how many zeros there will be on the righthand side of the answer to these math problems?

$25 \times 5 \times 2 \ =$

$32 \times 50 \times 12 \times 15 \times 90 \ =$

8d. Rewrite this as a power of 2:

$$2^{2008} + 2^{2008} + 2^{2008} + 2^{2008}$$

8e. Rewrite this as 3 times some power of 2:

$$2^{2013} - 2^{2011}$$

8f. Try a similar approach here:

$$2^{2010} + 2^{2009}$$

Now ... Let's see what you can do!

1. Simplify to a power of 3: $3^3 + 3^3 + 3^3 =$

2. Simplify to a power of 15: $3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$

3. Solve to a number: $2^5 \cdot 5^2$

4. Solve to a number: $2^2 + 2^1 + 2^0$

5. Simplify to a power of 10: 1,000,000,000

6. What ones digit do you get when you multiply out 123^{125}?

7. Simplify to a power of 2: $2^5 + 4^3 + 5^2 + 6^1 + 7^0$

8. Each of the numbers 2, 3, 4 and 5 are randomly assigned to a, b, c and d. What is the largest possible value of:

$$a^b + c^d$$

9. What is the last digit (the ones digit) for this sum, if you write it out and add it all up?

$$7^1 + 7^2 + 7^3 + \cdots + 7^{2010}$$

10. Solve this, and then tell me the sum of the digits of your answer. For instance, if your answer is 19, the sum of the digits would be 20; and if you answer is 145, the sum is 10.

$$10^{2010} - 1$$

11. What is the remainder when 3^{777} is divided by 7?

12. The integer 64 is both some number squared (8) and some number cubed (4). Find the next integer that has this same property.

13. How many 30s must be added together to get a sum equal to 30^3?

14. What is the remainder when 2^{100} is divided by 10?