Powers and Exponents (Part Two)

November 2019

1. Remember, exponents are just a shorthand.

1a. $4^5 = \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4} \cdot \underline{4}$

1b. $4^5 \cdot 4^2 =$ 4^7

1c. $5^3 \div 5^1 =$ 5^2

1d. $(3^2)^3 =$ 3^6 $\leftarrow$ $\underline{3 \cdot 3} \cdot \underline{3 \cdot 3} \cdot \underline{3 \cdot 3}$

1e. $(4^{10})^3 =$ 4^{30}

2. We can compare numbers easily if they have the same BASE.

2a. Which is bigger: 2^5 2^9 2^3

2b. Which is bigger: 13^5 13^9 13^3

2c. Which is bigger: $13^3 \cdot 13^2$ $13^6 \cdot 13^3$

3. We can compare numbers if we can cause them to have the same BASE.

3a. Which is bigger: 8^2 2^4 $(2^3)^2 = 2^6$

3b. Which is bigger: 4^3 16^9 $(4^2)^9 = 4^{18}$

3c. Which is bigger: 81^4 3^5 $(3^4)^4 = 3^{16}$

3d. Which is bigger: $125^1 \cdot 5^3$ $5^2 \cdot 5^5$

$5^3 \cdot 5^3 = 5^6$

4. We can compare numbers easily if they have the same EXPONENT.

4a. Which is bigger: 5^{15} 6^{15}

4b. Which is bigger: 8^{16} 9^{16}

4c. Which is bigger: $3^5 \cdot 5^5$ $4^5 \cdot 8^5$ 15 or 32

4d. Which is bigger: $9^6 \cdot 2^6$ $3^6 \cdot 7^6$ 18 or 21

5. We can compare numbers if we can cause them to have the same exponent.

5a. Which is bigger: 8^3 5^9 $(2^3)^3 = 2^9$

5b. Which is bigger: 16^2 3^8 $(2^4)^2 = 2^8$

6. Sometimes we can easily compare numbers even if the bases and exponents are different.

6a. Which is bigger: 8^3 7^2 $8 > 7$ $3 > 2$

6b. Which is bigger: 9^{14} 6^{11} $9 > 6$

$14 > 11$

7. When numbers are multiplied together, sometimes we can compare them most easily be re-writing them down using "factors" and exponents.

$9 \times 81 = 3^2 \cdot 3^4 = 3^6$

7a. Which is bigger: 9×81 $\quad$ 18×12

$18 \times 12 = 3 \times 6 \times 6 \times 2$

$= 3^3 2^3$

7b. Which is bigger: 25×36 $\quad$ 81×20

$2^2 3^2 5^2$ $\quad$ $2^2 3^4 5$

8. And sometimes there are cool puzzles to think through.

8a. Without doing the math, can you predict which of these is <u>*greater*</u>?

$(7^7 + 8^8)$ $\quad$ 9^9

$9^9 = 9 \cdot 9^8$

$= 9^8 + 9^8 + 7 \cdot 9^8$

Bigger than 7^7

Bigger than 8^8

8b. Without doing the math, can you predict which of these is <u>*greater*</u>?

2^{333} $\quad$ 3^{222}

$(2^3)^{111}$ $\quad$ $(3^2)^{111}$

8 $\quad$ 9

8c. The number 100 has 2 zeros on the righthand side of the number. The number 2,800 has 2 zeros on the righthand side of the number. The number 13,400,000 has 5 zeros on the righthand side of the number. Without doing the math, can you predict how many zeros there will be on the righthand side of the answer to these math problems?

$25 \times 5 \times 2 =$ Look for 2×5

1

$32 \times 50 \times 12 \times 15 \times 90 =$ 4 5's are in there

2 2×5×5 2 5 5×2 Plenty of 2's

So: 4

8d. Rewrite this as a power of 2:

$$2^{2008} + 2^{2008} + 2^{2008} + 2^{2008}$$

$4 \cdot 2^{2008} = 2^2 \cdot 2^{2008} = 2^{2010}$

8e. Rewrite this as 3 times some power of 2:

$$2^{2013} - 2^{2011}$$

$2^2 \cdot 2^{2011} - 2^{2011}$

$4 \cdot 2^{2011} - 2^{2011} = \boxed{3 \cdot 2^{2011}}$

8f. Try a similar approach here:

$$2^{2010} + 2^{2009}$$

$2 \cdot 2^{2009} + 2^{2009} = \boxed{3 \cdot 2^{2009}}$

Now ... Let's see what you can do!