Lesson 6: Greatest Common Divisor

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Definition 1.
The *greatest common divisor* (GCD) of two positive integers a, b is the biggest positive integer d such that $d \mid a$ and $d \mid b$. We denote the GCD of a and b by $\gcd(a, b)$.

Problem 1.
Compute the GCD of 47124 and 11050.

Problem 2.
a) Let a, b be positive integers, and $r > 0$ be the remainder of a when divided by b. Then $a = bq + r$ where q is an integer. Let S be the set of all common divisors of a and b, and let T be the set of common divisors of b and r. Prove that $S = T$.
Hint: if you want to show that two sets are equal, you need to show that every element of S is also an element of T and vice-versa.

b) Prove that $\gcd(a, b) = \gcd(b, r)$.

Problem 3.
Show that the fraction
$$\frac{12n + 1}{30n + 1}$$
is irreducible for all positive integers n.

Problem 4.
Can the GCD of two distinct positive integers be bigger than their difference?