Powers!

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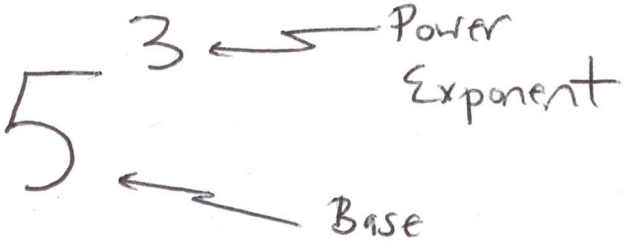
EXPONENTS ARE ... POWERFUL!

1. Let's start by talking like a mathematician. We need to introduce two words:

   "**base**"

   "**power**"



2. Most of the time, exponents are just a shorthand for multiplication. See if you can write out the following examples using *multiplication* instead of *exponents*.

$2^4 = 2 \times 2 \times 2 \times 2$

$3^4 = 3 \times 3 \times 3 \times 3$

$5^3 = 5 \times 5 \times 5$

$10^4 = 10 \times 10 \times 10 \times 10$

$3^{25} =$

No thank you!

Now do you see why exponents help?

3. In those examples, the *exponent* told you how many times to use the *base* in your multiplication problem. But there is one special case we need to just memorize.

$8^0 = 1$

$5^0 = 1$

$16^0 = 1$

This is just a rule, like "knew" is spelled with a silent "k". There is no logic to it, unlike the prior question.

4. So what makes exponents exciting?

4a. Suppose you wanted to travel from planet Earth to planet Saturn. Roughly how many miles would you need to travel? (It's okay to guess.) Can you write that number down with exponents? Without? Which is easier?

The point: I can write huge numbers easily and clearly with exponents, like 10^{14}.

4b. I have a cold. I came to school anyway and caused two friends to get sick. The next day, my two friends came to school and each infected two other friends. If this pattern continues, how many kids will get sick on Friday? What about the next Friday? The one after that? (Anyone have a tissue?)

SUN MON TUES
2 4 8 ...

On day X, we infect 2^X kids.

The point: exponents can describe real world behavior elegantly!

DON'T GET TRICKED!

1. $2 \cdot 2^5 = 2^?$ 2^6 because $5+1$
2. $4^5 \div 4 = 4^?$ 4^4
3. $2^5 \cdot 2^5 = 2^?$ 2^7
4. $8^5 \cdot 2^3 = 8^?$ $2^3 = 8$ so $8^5 \cdot 8 = 8^6$
5. $8^5 \cdot 2^3 = 2^?$ $8 = 2^3$ so $2^{15} \cdot 2^3 = 2^{18}$
6. $3^5 \cdot 3^3 = 3^?$ 3^8
7. $3^2 + 3^5 = 3^?$ Trick! this is NOT 3^7
8. $8^4 - 8^2 = 8^?$ Trick! this is NOT 8^2
9. $100 = 2^?5^?$ $100 = 10 \cdot 10 = 2 \cdot 5 \cdot 2 \cdot 5 = 2^2 5^2$
10. $72 \times 81 = 3^?2^?$ $72 = 9 \cdot 8 = 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2$
11. $32 \times 45 = 36 \times ??$ $81 = 9 \cdot 9 = 3 \cdot 3 \cdot 3 \cdot 3$

so $72 \cdot 81 = 3^6 2^3$

$32 \cdot 45 = 4 \cdot 8 \cdot 5 \cdot 9$

$= 4 \cdot 9 \cdot 8 \cdot 5$

$= 36 \cdot 40$

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