Lesson 3: More tilings and some algebra.

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Problem 1.
What is the biggest number of 1×4 rectangles that can be fit into a 6×6 square without overlaps?
Hint: In the diagonal coloring with four colors, one of the colors has 8 squares. Then we can fit at most 8 rectangles, An example is easy to construct.

Problem 2.
Ninety nine 2×2 squares were cut out of a 29×29 board. Prove that it is possible to cut out at least one more.
Hint: consider the following coloring: a square (i, j) is black if $i \not\equiv 2 \pmod 3$ and $j \not\equiv 2 \pmod 3$. Visually this looks like 2×2 squares separated by single rows and columns. Then every cut out 2×2 square touches at most one black square, and there are 100 black squares.

Problem 3.
Prove that 8999999 is not a prime number.
Hint: $8999999 = 9000000 - 1 = 3000^2 - 1 = 2999 \cdot 3001$.

Problem 4.
Expand $(a + b - 2c)^3$.

Problem 5.
Factor the following polynomials:

a) $ac + ad + bc + bd$.

b) $ac + bc - ad - bd$.

c) $1 + a + a^2 + a^3$.

d) $1 + a + a^2 + a^3 + \ldots + a^{14}$.
Hint: $(1 + a + a^2 + a^3 + a^4)(1 + a^5 + a^{10})$

e) $x^4 - x^3 + 2x - 2$.