Algebra test

Konstantin Miagkov

May 5, 2019

Problem 1.
Find all values of x and y such that $x - y = 5$ and $5x - 3y = 9$.
Answer: $x = -3, y = -8$.

Problem 2.
Expand $(a - b + 2c)^2$.
Answer: $a^2 + b^2 + 4c^2 - 2ab - 4bc - 4ac$.

Problem 3.
a) Prove that in an isosceles triangle $\triangle ABC$ with $AB = AC$ the median and the altitude out of point A coincide.

Proof. Let AM be the median of $\triangle ABC$. Then $AM = AM$, $BM = MC$, $AB = AC$ and thus by the SSS test we have $\triangle ABM = \triangle ACM$. Then $\angle AMB = \angle AMC$, and those angles add up to $180°$. Thus $\angle AMC = \angle AMB = 90°$, and AM is the altitude as well. □

b) Prove that in an isosceles triangle $\triangle ABC$ with $AB = AC$ the median and the angle bisector out of point A coincide.

Proof. If AM is the median, we have already established that $\triangle ABM = \triangle ACM$. Then $\angle BAM = \angle CAM$, and so AM is also an angle bisector. □

Problem 4.
a) Factor $a^2 - b^2$.
Answer: $(a - b)(a + b)$.

b) Factor $a^3 - b^3$.
Answer: $(a - b)(a^2 + ab + b^2)$.

c) Factor $a^n - b^n$ where n is a positive integer.
Answer: $(a - b)(a^{n-1} + a^{n-2}b + \ldots + b^{n-1})$.

d) Factor $a^n + b^n$ where n is an odd positive integer.
Answer: $(a + b)(a^{n-1} - a^{n-2}b + \ldots + b^{n-1})$.

Problem 5.
Let y, z be two different solutions to the equation $x^2 + 8x + 13 = 0$. What is the value of

$$\frac{yz}{y + z}$$

Proof. By Vieta's theorem, $yz = 13$ and $y + z = -8$. Then $yz/(y + z) = -13/8$. □