More Problem Solving*

November 21, 2010

1. What digit does the star represent?

$$
\begin{array}{r}
2 \\
\star 7 \\
\star \\
+\ 21 \\
\hline
6\star
\end{array}
$$

2. In a number, the first digit is greater than the second digit by 2, and the second digit is greater than the third by 4. The third digit is even. What number is this?

3. Kim takes 25 minutes to walk to her favorite clothing store. If she leaves her house at 4:05 P.M., she will get to the store a half hour after it closes. If she leaves at 7:45 A.M., she will get there 20 minutes before the store opens. What hours is the store open?

4. The Math Circle Hotel has rooms that can hold 2 or 5 guests. If the hotel can hold a total of 41 guests, and has exactly 3 two-person rooms. How many five-person rooms does the hotel have?

5. Complete the statement below with the smallest possible number that will make it true.

The sum of two different numbers is 7. At least one of the numbers is less than __.

6. Place each of the following signs exactly once between the digits below to make an equation: $+, -, =$. (Note: the signs do not need to be used in that order)

1 0 5 2 1 6

7. A kangaroo noticed that each winter, he gains 5 pounds of weight, and each summer he loses 4 pounds. During the spring and the fall, his weight does not change. If the kangaroo weighed 100 pounds in the Spring of 2009, how much did he weigh in Fall of 2005?

8. Karen has \$5 and goes into a stationary store which sells notebooks for 85 cents and pencils for 20 cents. She needs to buy 5 notebooks for her class. How many pencils at most can she also buy at this store?

9. There are 9 streetlights on one side of an alley in the park. The distance between neighboring streetlights is 7 meters. Gregory walked from the first lantern to the last lantern. How many meters did he walk?

10. What digit is missing in the equation below?

$$4 + 43 + 11 + 12 = 90$$

11. To open a safe, a 5-digit code must be used. How many possible codes are there if it is known that the code only contains the numbers 1, 2, 3, 4 and 5, and each of those numbers is used only once in the code.

12. To open a different safe, a 4-digit code is needed. How many possible codes if it is known that only the numbers 1, 2, and 3 are used in the code in the following way: The numbers 1 and 3 are used one time each, and the number 2 is used twice.

13. Shannon solved 22 problems in the first two days of school. On the third day, she solved twice as many problems as on the first day. This is 8 problems more than she solved on the second day. How many problems did she solve on the first day?

14. What number needs to be written in the shaded cloud in order to get the number in the last cloud as a result of operations shown in the picture?
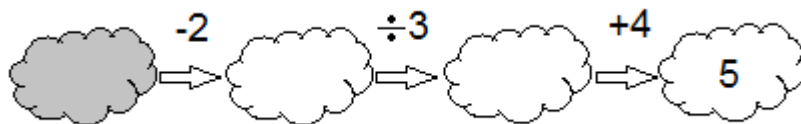
15. What is the smallest number that is bigger than 2007 and the sum of whose digits equals the sum of the digits in 2007?

16. A string was cut 5 times into equal pieces. If each of the pieces cut from the original string is exactly 3 centimeters long, how long was the original string?

17. We name a number as *palindromic* if it doesn't change after its digits are written in reverse order. Some examples of such numbers are 1331 and 24642.

   (a) Give an example of a 7-digit palindromic number.

   (b) A car's odometer shows the car has driven $15,951$ miles. What will be the next palindromic number that will show up in the odometer? After how many more miles will that number show up for the first time?

18. Each of the cards below has a letter on one side and a number on the other side.

(a) Rosalie made the following statement:

> If a card has the number 2 on one side, then it will have the letter J on the other side.

Which of the cards below would you need to flip order to find out whether Rosalie's statement is true or false?

| 5 | 2 | J |
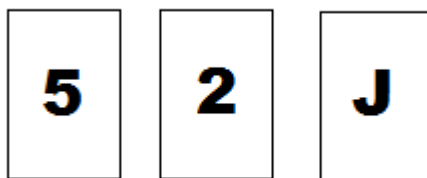
(b) Alex made the following statement:

> If a card has the letter A on one side, then the number on the other side of the card will be even.

Which of the cards below would you need to find out whether Alex's statement is true or false?

| 1 | Q | 8 | A |
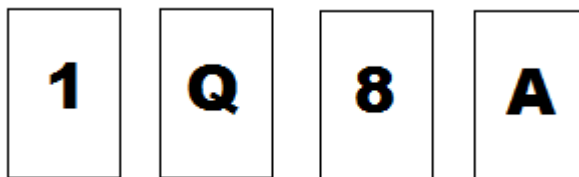
19. Daniel likes to add 6, Aadi likes to multiply by 3, and Tim likes to subtract 3.

    (a) In what order should they perform their favorite operations, each only once, so that starting with the number 4, they end with the number 9?

    (b) In what order should they perform their favorite operations, each only once, so that starting with the number 2, they end with the number 9?

20. Complete the statement below with the smallest possible number that will make the statement true.

    The sum of two different numbers is 25. At least one of the numbers is less than __.

    (a) 18   (c) 25
    (b) 14   (d) 13

21. There is one digit missing in the equation below. What digit is missing?

$$3 + 5 + 23 + 37 = 96$$

22. At a round table, there are chairs placed with the same distance between them. They are numbered subsequently: 1, 2, 3, ... Peter is sitting on chair number 11, directly across from Chris, who is sitting in chair 4. How many chairs are there at the table?