

RUSSIAN PEASANT MULTIPLICATION

JUNIOR CIRCLE 11/6/2011

Russian Peasant Multiplication is a similar procedure to Egyptian Multiplication. Suppose that you have to multiply two numbers (e.g., 21 and 18). The basic operation for them was multiplying or dividing a number by 2. They reduced all other multiplication problems to it. Here is how they would start multiplying 21 by 18 (in modern notation):

21	18
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21	18
10	36
5	72
2	144
1	288

Here is what to do to complete the multiplication

- (1) In the first column, take the first number and continue dividing it by 2 until we reach 1. If there is a remainder, drop it. (E.g. $21/2 = 10$ with remainder 1, so we just write 10.) Keep dividing until we end up at 1.
- (2) In the second column, we keep doubling the second number (in this case, 18).
- (3) After that, we choose the rows with odd numbers in the first column. (in our example, we chose the first, third, fifth and seventh row)

Finally, all there is to do at this point is to add the marked numbers in the second column:

		1	8
		7	2
+	2	8	8
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	3	7	8

Thus, the result of the multiplication is 378.

Multiply 13×22 using Russian Peasant Multiplication.

- Steps:
- 1) Create two columns
 - 2) In the first column, take the first number and continue dividing it by 2 until we end up at 1.
* If there is a remainder, drop it
 - 3) In the 2nd column, keep doubling the second number.
 - 4) Mark the rows with odd numbers in the first column. (✓)
 - 5) Add together the numbers of the second column of the marked rows.

13×22 :

	<u>13</u>	<u>22</u>
✓	13	22
	6	44
✓	3	88
✓	1	176

$$\begin{aligned}
 & - 13 \div 2 = 6.5 \quad * \text{Drop } .5 \text{ on } 1 \text{ (the Remainder)} \\
 & - 6 \div 2 = 3 \\
 & - 3 \div 2 = 1 \text{ Remainder } 1 \\
 & \quad \quad \quad 1 \checkmark
 \end{aligned}$$

$$\begin{array}{r}
 176 \\
 88 \\
 + 22 \\
 \hline
 \underline{\underline{286}}
 \end{array}$$

$$13 \times 22 = 286 \checkmark$$

(1) Using what you notice from the example do the following:

(a) Rewrite each term in the sum: $18 + 72 + 144$ using the result from question 4:

- $18 = 1 \cdot 18$ or 1×18

- $72 = 4 \cdot 18$ or 4×18

- $288 = 16 \cdot 18$ or 16×18

(b) Rewrite the whole sum:

- $18 + 72 + 288 = (1 \cdot 18) + (4 \cdot 18) + (16 \cdot 18)$

(c) What do you notice? Can you simplify this expression by factoring out 18?

I notice that each of the terms in the sum has 18 in common.
Therefore, we can simplify the sum by factoring out 18.

$$(1 \cdot 18) + (4 \cdot 18) + (16 \cdot 18) = 18(1 + 4 + 16) = \underline{18 \cdot 21}$$

Fact. Note that in this algorithm we are starting with the lowest power of 2 present in the number (E.g. 21)

To understand how Russian Peasant Multiplication works, let's learn to write numbers as sums of powers of 2 (without repetition) *starting with the smallest power of 2*.

Is this row used?	21	18	Multiplication Factor
1	21	18	$= 18 \cdot 1 : 2^0 = \underline{1}$
0	10	36	$= 2 \cdot 18 : 2^1 = \underline{2}$
1	5	72	$= 4 \cdot 18 : 2^2 = \underline{4}$
0	2	144	$= 8 \cdot 18 : 2^3 = \underline{8}$
1	1	288	$= 18 \cdot 16 : 2^4 = \underline{16}$

- In the column labeled "Is this row used?" write 1 if the row is used and 0 if it is not used in Russian Peasant Multiplication.
- In the column labeled "Multiplication Factor" write the power of 2 by which 18 is multiplied in this row. (Do this for all rows.)
- What do the numbers in the column "Multiplication Factor" and marked rows add up to: The rows marked with 1 are used and the column "Multiplication Factor" of the marked rows ~~are~~ contain the following values:
1, 4, and 16.

The sum of these values are: $1 + 4 + 16 = 21$.

Note: 21 is our first number.

Use this method to multiply:

(1) 27×23

	27	23	
1	27	23	1
1	13	46	2
0	6	92	4
1	3	184	8
1	1	368	16

Sum of
3rd Column
Values of
Rows Used:

$$\begin{array}{r} 23 \\ 46 \\ 184 \\ + 368 \\ \hline 621 \end{array}$$

$27 \times 23 = 621$

For Q.4: Sum of Powers of 2

$1 + 2 + 8 + 16 = 27$
* First number

(2) 33×22

	33	22	
1	33	22	1
0	16	44	2
0	8	88	4
0	4	176	8
0	2	352	16
1	1	704	32

$$\begin{array}{r} 22 \\ + 704 \\ \hline 726 \end{array}$$

$33 \times 22 = 726$

For ~~Q.4~~ Q.4: Sum of Powers of 2

$1 + 32 = 33$
* First number

(3) 19×45

	19	45	
1	19	45	1
1	9	90	2
0	4	180	4
0	2	360	8
1	1	720	16

$$\begin{array}{r} 45 \\ 90 \\ + 720 \\ \hline 855 \end{array}$$

$19 \times 45 = 855$

For Q.4: Sum of Powers of 2

$1 + 2 + 16 = 19$
* First number

(4) Which number are we writing as a sum of powers of 2? The first number or the second number?

We are writing the first number as the sum of powers of 2.

* See above for proof *

This gives us an algorithm of writing a number as a sum of powers of 2 starting with the smallest power. It is as follows:

-		-

- (1) At each step, divide the number you currently have by 2 (in the middle column).
- (2) If this number is even, write 0 in the left column. If the number is odd, write 1 in the left column.
- (3) In the right column, write the powers of 2, starting with $2^0 = 1$.
- (4) Circle the powers of 2 that are in the rows where the left column has 1.
- (5) The circled numbers in the right column add up to the number you started with.

Use the algorithm to write the following numbers as powers of 2 starting with the smallest power:

• $9 = 1 + 8$

• $14 = 2 + 4 + 8$

• $23 = 1 + 2 + 4 + 16$

• $44 = 4 + 8 + 32$

	9	
1	9	$2^0 = 1$
0	4	$2^1 = 2$
0	2	$2^2 = 4$
1	1	$2^3 = 8$

	14	
0	14	$2^0 = 1$
1	7	$2^1 = 2$
1	3	$2^2 = 4$
1	1	$2^3 = 8$

	23	
1	23	$2^0 = 1$
1	11	$2^1 = 2$
1	5	$2^2 = 4$
0	2	$2^3 = 8$
1	1	$2^4 = 16$

	44	
0	44	$2^0 = 1$
0	22	$2^1 = 2$
1	11	$2^2 = 4$
1	5	$2^3 = 8$
0	2	$2^4 = 16$
1	1	$2^5 = 32$

RACE!

- (1) Represent each number below as a sum of powers of 2. One person uses the Egyptian method (which starts with the largest power of 2 first and multiplication by 2). The other person uses the Russian Peasant method (which starts with the smallest power of 2 first and division by 2).

(a) 26

Russian Peasant Multiplication

$$26 = 2 + 8 + 16$$

26	1	
0	26	1
1	13	2 ✓
0	6	4
1	3	8 ✓
1	1	16 ✓

Egyptian Multiplication

26	
1	✓
2	✓
4	✗
8	✓
16	✓

$$26 = 16 + 8 + 2$$

$$* 16 + 8 + 4 > 26$$

(b) 45

$$45 = 1 + 4 + 8 + 32$$

45	1	
1	45	1
0	22	2
1	11	4
1	5	8 ✓
0	2	16
1	1	32 ✓

45

45	
1	✓
2	✓
4	✓
8	✓
16	✓
32	✓

$$35 < 45 ✓$$

$$35 + 16 > 45 ✗$$

$$35 + 8 < 45 ✓$$

$$35 + 8 + 4 > 45 ✗$$

$$35 + 8 + 2 = 45 ✓$$

$$45 = 35 + 8 + 2$$

(c) 52

$$52 = 4 + 16 + 32$$

52	1	
0	52	1
0	26	2
1	13	4 ✓
0	6	8
1	3	16 ✓
1	1	32 ✓

52

52	
1	✓
2	✓
4	✓
8	✓
16	✓
32	✓

$$32 < 52$$

$$32 + 16 < 52$$

$$32 + 16 + 8 > 52 ✗$$

$$32 + 16 + 4 = 52 ✓$$

$$52 = 32 + 16 + 4$$

(d) 68

$$68 = 4 + 64$$

68	1	
0	68	1
0	34	2
1	17	4 ✓
0	8	8
0	4	16
0	2	32
1	1	64 ✓

68

68	
1	✓
2	✓
4	✓
8	✓
16	✓
32	✓
64	✓

$$64 < 68$$

$$64 + 32 > 68 ✗$$
~~$$64 + 32 + 16 > 68$$~~
~~$$64 + 32 + 4$$~~

$$64 + 16 > 68 ✗$$

$$64 + 8 > 68 ✗$$

$$64 + 4 = 68 ✓$$

$$68 = 64 + 4$$

(e) Which one do you like better?

I like the Russian Peasant better for depending on whether the middle column contains an odd or even number, we are able to determine whether the corresponding power of 2 is included in the sum.

- (2) Is Russian Peasant Multiplication easier or harder than Egyptian Multiplication? I found the Russian Peasant Multiplication to be easier than Egyptian Multiplication because you know for sure whether a power of 2 is included in the sum whereas in the Egyptian Multiplication, there is a trial and error component.
- (3) Compare Russian Peasant Multiplication, Egyptian Multiplication and regular, long multiplication. What are the key differences? Which requires you to remember the most facts? Use the column below to write the pros for each type of multiplication.

Russian Peasant Multiplication	Regular, Long Multiplication	Egyptian Multiplication
<ul style="list-style-type: none"> * Starts with the smallest power of 2 * Utilizes division by 2 * Using odd and even numbers, we can determine whether a power of 2 is included in the sum * Requires one to remember the most number of facts. 	<ul style="list-style-type: none"> * Requires the memorization or knowledge of the multiplication table. * Fastest method to find the answer. 	<ul style="list-style-type: none"> * Starts with the largest power of 2 first and multiplies * Utilizes multiplication by 2 * Trial and error component