

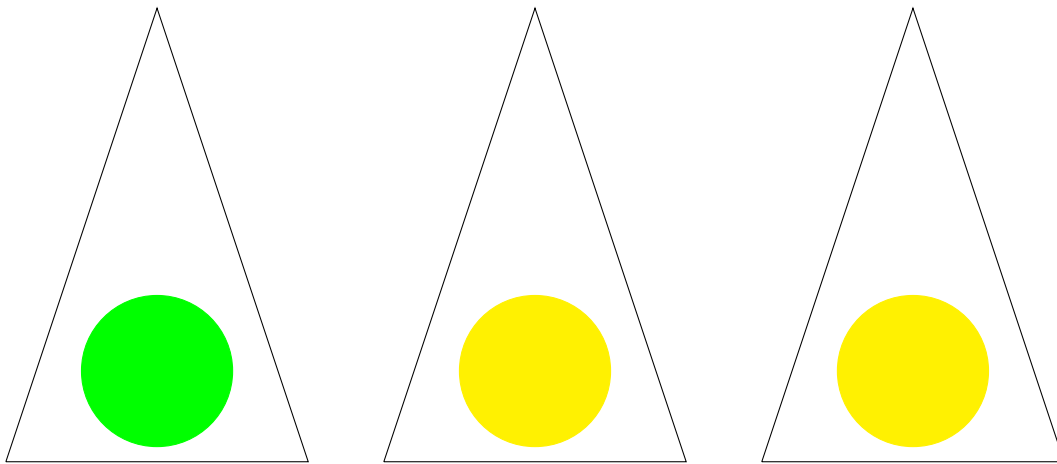
SOLUTIONS

LAMC Junior Circle

Jan 12, 2014

Logic Skills: Hats and Doors

A part of wizardry training is developing logic skills. In the following exercises, two students will be presented with three wizard's hats, one with a green label and two with yellow labels.



The students will be asked to close their eyes. One hat will be hidden, the others will be put on the students' heads. The students will open their eyes, and then be asked, "Who knows what color hat he/she has on? If so, which color is it?" The students can figure out the color hat he/she is wearing only by observing the hat of the other student. A student is not allowed to look at his/her own hat. This kind of cheating will be punished by turning the cheat into a toad!

Problem 1 *Kaley and Emmanuelle are given two hats. Kaley gets the green-labeled hat, and Emmanuelle gets a yellow-labeled one.*

a) Cory asks them what color hat they have on. What are Kaley and Emmanuelle going to say? Explain.

Kaley: "I don't know"

Emmanuelle: "Yellow!"

Kaley sees a yellow hat so she knows that Kaley either has a yellow or green hat.

Emmanuelle sees a green hat and knows Emmanuelle must have a yellow hat.

Note: Assuming they reply at the same time

b) After hearing Emmanuelle's answer, can Kaley now figure out which color hat she has on? Explain why or why not.

Yes, Kaley can figure out Kaley has a green hat. If Kaley had a yellow hat, then Emmanuelle would not know if Emmanuelle had a green or yellow hat. However, since Emmanuelle knows Emmanuelle is yellow, that must mean Emmanuelle is looking at a green hat

Problem 2 *This time, Kaley and Emmanuelle are both given the yellow-labeled hats.*

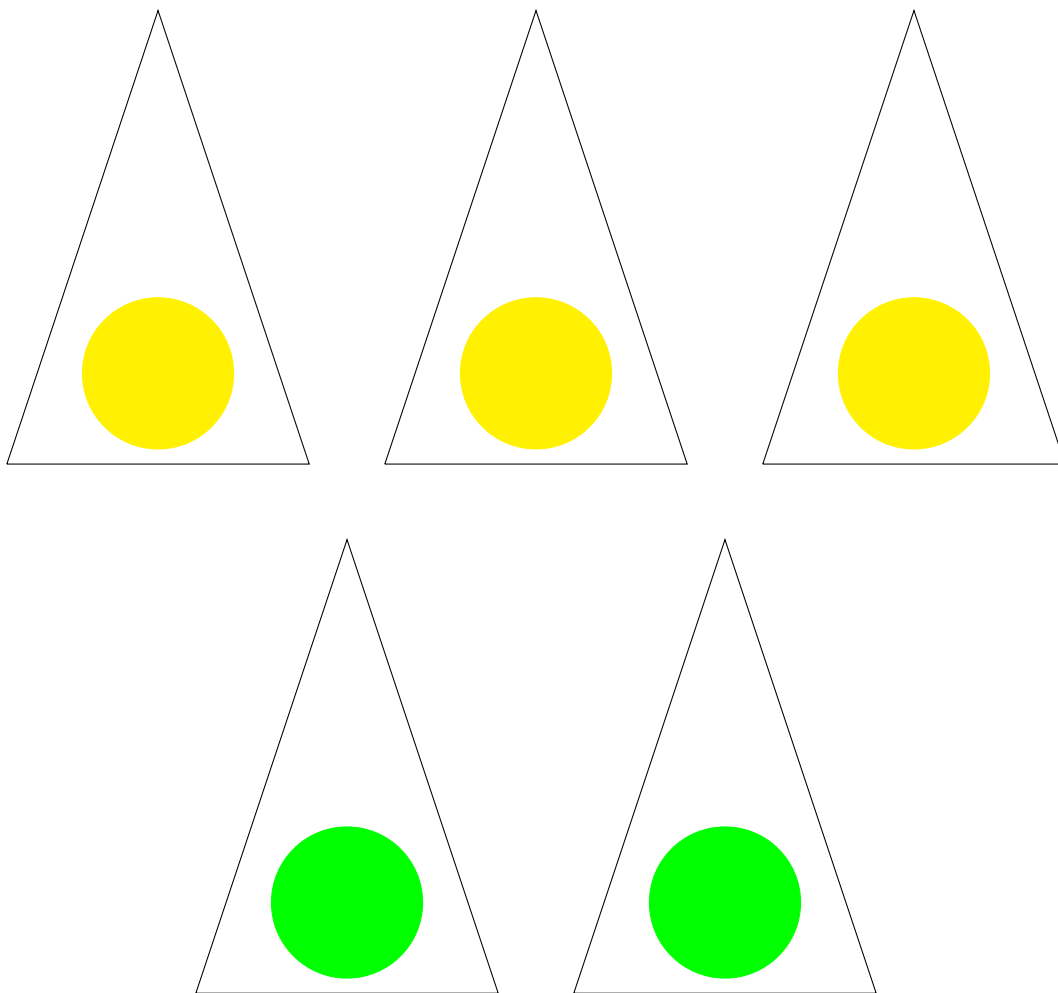
a) Cory asks them what color hat they have on. What are Kaley and Emmanuelle going to say? Explain.

They will both say they don't know because they each see a yellow hat and know the hat on their own head can be either yellow or green (again assuming they answer simultaneously).

b) Now that they heard each other's answers, can either of them figure out the color of their hat? How?

Yes. They both can guess they have yellow hats. This is because since they each know the other doesn't know which hat they are wearing on their own head. This means they know the other is also looking at a yellow hat and can deduce they're both wearing a yellow hat.

As the training progresses, three students will be chosen and presented with five hats, three with yellow dots and two with green. Once again, the students will be asked to close their eyes. Two hats will be hidden, three will be put on the students' heads. The students will be asked to open their eyes and to figure out the color of their hat labels by observing those of the other students.



Problem 3 *Kaley, Emmanuelle, and Lucy are given the hats. Kaley gets a yellow-labeled hat, Emmanuelle and Lucy get the hats with green labels.*

a) What will each person say when Cory asks them for the color of their hats? Explain.

Assuming they answer simultaneously:

Kaley: "Yellow!"

Emmanuelle and Lucy: "I don't know"

Kaley sees two green hats so Kaley knows Kaley must have a yellow hat. Both E and L see a yellow and green hat so they don't know if they have a yellow or green.

b) After hearing each person's answer, can others figure out which color hat they have on? How?

Yes. Since Kaley knows she has a yellow hat, that means she must see two green hats. Hence, E and L will say they have a green hat.

Problem 4 *This time, Kaley and Emmanuelle get yellow-labeled hats while Lucy gets the hat with a green label.*

a) How will each person respond to Cory's question? Explain.

They will all say they don't know. This is because for each, the open options left still include green and yellow. K and E both see yellow and a green, and L sees two yellow hats. However, this means they all cannot be certain of their respective color.

b) Now that they have heard each other's answers, who can say the color of their hat and who can't? Explain why or why not. (Hint: What would happen if one assumes that she is wearing the green hat?)

K and E can both say they have yellow hats. This is because they both see a green and yellow hat. If their own hat was green, that means the one who has a yellow hat (E or K) should be able to guess correctly they have a yellow hat. But because no one was able to do so, E and K are able to say that they have a yellow hat.

L still does not have enough information. If she has a yellow hat, that means E and K would also see two yellow hats and be uncertain. But if she had a green hat, K and E would both see a yellow and green hat and still not be able to make a conclusion

c) After hearing everybody's answers, who can now figure out the color of their hat? Explain. (Hint: What would happen if Lucy assumes she is wearing the yellow hat?)

Now Lucy can finally figure out she has a green hat. This is because if she has a yellow hat, K and E should've been in the same position as her and be unable to figure out their own hat color in the second round. But because she has a green hat, based on the logic in part b, K and E were able to figure out their own hat colors.

Problem 5 *Finally, all the three students are given the yellow-labeled hats.*

a) How would each person respond to Cory's question? Explain.

They would all say they don't know because they all only see two yellow hats and cannot make any solid conclusions about the color of their own hat

b) *After hearing everybody's answer, who can say the color of their hat and who can't? Explain why or why not. (Hint: What would happen if one of them assumes she is wearing a green hat?)*

Nobody. Everyone still can't figure out their own color. If one assumes they have a green hat, that means following problem 4 the other two in this round should be able to answer their color, but not the person who assumed he/she has the green hat.

c) *After hearing each person's answer, Lucy tries to figure out the color of her hat. She initially makes the assumption that she is wearing a green hat.*

i) *If Lucy was wearing the green hat, would anybody have immediately known which color hat they had on? Does this agree with what you wrote for part a?*

In the first round, Lucy knows the other two are Y and Y. Lucy herself does not know if her hat is G or Y. But if Lucy assumes that her own hat is G, then she would expect that neither K nor E would be able to guess either. After all, if Lucy had G, K would see G_Y, and thus K would not know if the right answer was GYY or GGY, and meanwhile E would see GY_ and now know if the right answer was GYY or GYG. This agrees with 5a: all three would say "I don't know" if Lucy had a green hat.

ii) *After hearing everybody's answer, would anybody be able to figure out the color of their hat? (Remember, Lucy is assuming that she has a green hat on.) Does this agree with what you wrote for part b? If the answer is not the same as part b, explain what this contradiction means.*

The second round, however, is more informative. If Lucy assumes she has a G, and she hears everyone say "I don't know" in the first round, then she will want to listen closely to the next round. After all, when it is K's turn to speak, K would be seeing "G_Y" and K would know that L did not see "GG_" because L did not yell out "Yellow!". So K would know that L saw "GY_" last round, and thus K would right now yell, "Yellow!". But if K is silent, then K must be seeing "Y__". That would explain why L was quiet last round: L saw "YY_" and so did not guess. K's silence thus tells Lucy that Lucy must not have G.

iii) *Does Lucy know which color hat she has on? Does everybody else know?*

Yes, in the third round, Lucy can now confidently say she has Y. And, by the same process, everyone else also knows they have Y also

There once was a king who designed the following way of punishing criminals. Convicted lawbreakers were given a choice between two doors. Behind each door, there could be either a hungry tiger or a treasure of gold, but not nothing or both. Sometimes, the king would be merciful and put gold behind both doors. However, he can be cruel and put tigers behind both doors. The king would also post some warnings on the doors and then let the criminals choose.

Problem 6 *The king took the prisoner to the doors. There was a sign on each door. The first read, “There is gold in this room and there is a tiger in the other.” The sign on the second door read, “There is gold in one of these rooms and in one of these rooms there is a tiger.” “Are the signs true?” asked the prisoner. “One of them is,” replied the king, “but the other is not. Now, make your choice, buddy!” Which door should the prisoner open? Why?*

The prisoner should take the second door to escape.

If the first sign was true, that implies that the second sign is true as well. So this is not the case. Therefore, only the second sign is true and the first sign is false, meaning the gold is in the second door.

Another way to solve the problem is to write down the four possibilities and test them. The four possibilities are GG, GT, TG, and TT. Applying the sentences to each, you see that only in the example TG do we find that one sentence is true while the other is false.

Problem 7 *For the second prisoner, the following signs were put on the doors. Door 1: at least one of these rooms contains gold. Door 2: a tiger is in the other room. “Are the signs true?” asked the prisoner. “They are either both true or both false,” replied the king. Which door should the prisoner choose? Why?*

If both signs are true, that means the first door has a tiger and the second contains gold. If both signs are false, that means less than one room contains gold, or in other words, none of the rooms contain gold and both contain tigers. But this makes the second sign true, and thus this cannot be the answer.

Hence, both signs are true and the prisoner should choose the second door.

Problem 8 *In this case, the king explained that, again, the signs were either both true or both false. Sign 1: either this room contains a tiger, or there is gold in the other room. Sign 2: there is gold in the other room. Does the first room contain gold or a tiger? What about the other room?*

We don't know about the first door, but the second door has gold.

If both signs are true, then this means that the first room has gold based on the second sign, thus implying the second room also has gold from the first sign.

If both signs are false, that means based on the second sign the first room does NOT contain gold, or in other words it contains a tiger. And if the first sign is false as well, then there must be gold in the other room.

Thus the first room can have either a tiger or gold, but the second door must have gold

Once upon a time, in a land far, far away there lived a very beautiful princess, the only daughter of a very evil king. Some day, a handsome and very smart prince from a neighboring kingdom came to pay them a visit.

The princess and prince fell in love with each other and asked the king for a permission to marry. The evil king didn't want his daughter to leave. Instead of blessing the marriage, he ordered to put the prince in jail and to prepare for his exile.

The princess begged the king not to kill the prince and finally he agreed. He told the prisoner, "At her Highness's request, I will give you a chance. Tomorrow you will be brought to my court. You will have to pull a lot. I will put two pieces of paper in the box. One will read MARRIAGE, the other will read EXILE. Whatever piece you pull out, it will be your destiny."

The king was a very evil man. He ordered his minister to write EXILE on both pieces of paper. The princess overheard the king's order and found a way to warn the prince.

Problem 9 *What should the prince do to survive?*

Hint: kings do not like public embarrassment.

Somehow the prince picks the first piece of paper, but before the revealing the slip, it gets destroyed, so that the prince says that means the one he chose must be the opposite of the remaining slip. Since the second slip says "EXILE", the first slip supposedly should say "MARRIAGE"