1 The abacus

Materials for the lesson and homework: a couple of pencils, an eraser, a pencil sharpener; an abacus with ten beads on a wire. We suggest not to use a software abacus available on most smartphones, tablets, etc.

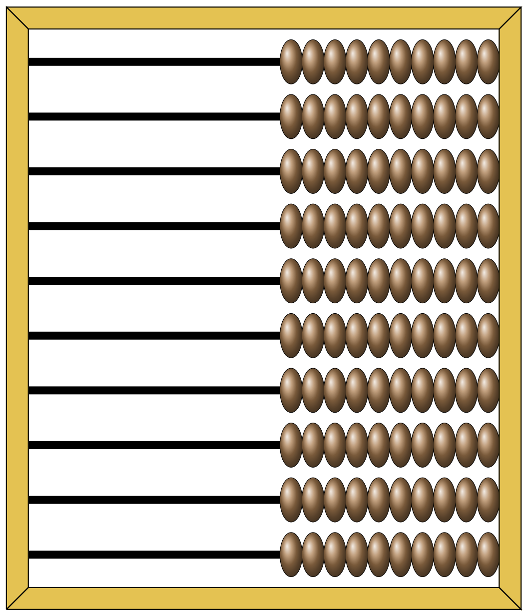
Warm-up

Problem 1.1 *Insert the signs + and − between the digits below to make a correct statement. Try to find as many solutions as possible.*

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 = 100$$

1.1 Lesson

A great tool for teaching children addition and subtraction of multi-digit numbers is the Russian *abacus*, a deck with multiple wires and ten beads on each wire as on the picture below.



Abaci[1] were used in the societies pre-dating Ancient Greece, such as Sumerian, Egyptian, Hebrew, and others. The word *abacus* originates from the Hebrew *ābāq*, meaning *dust*. The computers of the ancient world were first made of sand grooves filled with stones, hence the name.

Forgotten in Europe by the end of the 16th century, the abacus was reintroduced by a French mathematician, Jean-Victor Poncelet. A military engineer in the Napoleon army, Monsieur Poncelet was captured by the Russian troops in 1812 and released in 1814. He brought the

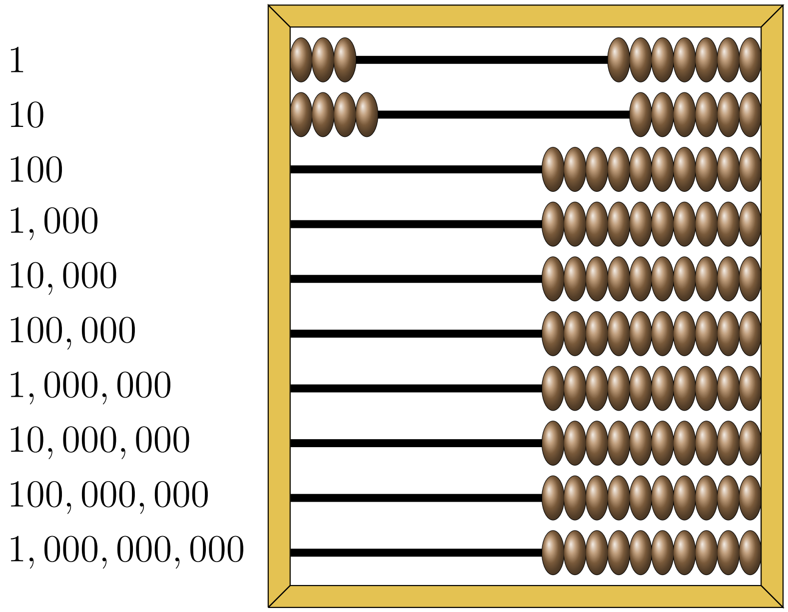
[1] Plural for *abacus*.

abacus to France from his Russian captivity and began using it as a teaching tool. In Russia itself, the abacus was a standard computational device on every counter, from a bank to a grocery store, until the middle of the 1980s, when it eventually got replaced by electronic calculators.

A picture from Smithsonian Libraries

https://library.si.edu/image-gallery/74037

We call the abacus's top wire the first, the second from the top wire the second, etc. We use the beads on the first wire to count ones, the beads on the second wire to count tens, the beads on the third wire to count hundreds and so forth. For example, the number 43 is shown on the picture on the next page. 43 is made of three ones and four tens. Therefore, the number is represented by three beads on the first wire of the abacus and by four beads on the second wire.



We call the numbers 1, 10, 100, 1000, etc., *basic decimal numbers*. Note that the name is not standard! It is specific for this book.

The symbols we use to write down numbers, like 43, show us how many basic decimal numbers to take. As we have observed, 43 = four 10s and three 1s. We can write this decomposition in a table.

studied number	*basic decimal numbers*				
	10000	**1000**	**100**	**10**	**1**
43				4	3

Note that the abacus and the table show the same thing!

The numeral system currently in use by humanity is *decimal place-value*. The word *decimal* means that we use ten digits to make numbers. The words *place-value* mean that a value of a digit in a number depends on its position in the number. For example, the value of the digit 4 is four tens in the number 43, but it is four ones in the number 504.

Problem 1.2 *Use the chart below to show the breakdown of the number* 504*. Then make the number on the abacus.*

studied number	*basic decimal numbers*				
	10000	**1000**	**100**	**10**	**1**
504					

Problem 1.3 *Make the following numbers on the abacus and show your teacher one after another:* 3*,* 10*,* 12*,* 47*,* 81*,* 100*,* 693*,* 963*.*

To understand the workings of the abacus and of the place-value decimal system, it is very important to realize the following.

- 10 = ten 1s. Therefore, one bead on the second wire of the abacus is equivalent to all the ten beads on the first wire.

• 100 = ten 10s. Therefore, one bead on the third wire is equivalent to all the ten beads on the second wire.

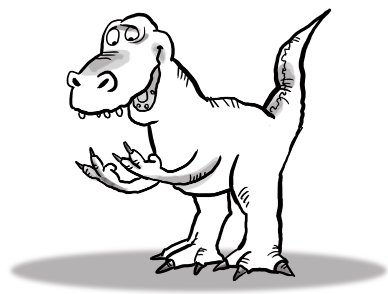
• 1000 = ten 100s. Therefore, one bead on the fourth wire is equivalent to all the ten beads on the third wire, and so forth.

As you can see the number 10 is fundamental for the decimal system. It is called the system's *base*.

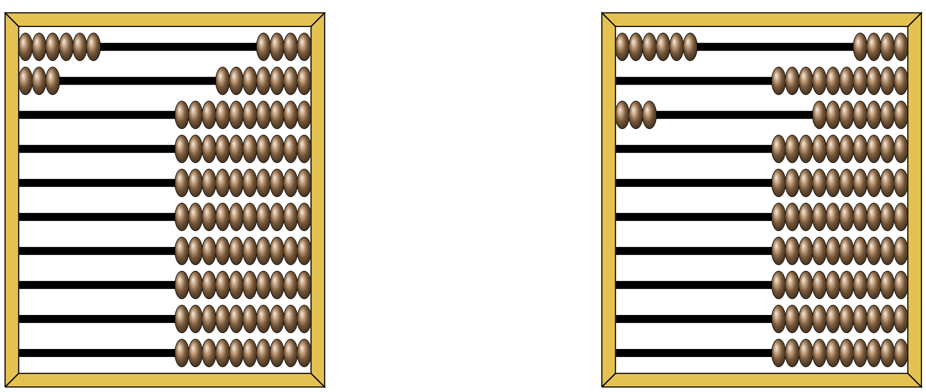
Question 1.1 *Did you know that the second meaning of the word "digit" is "finger"?*

Question 1.2 *Why do you think we use ten digits in math?*

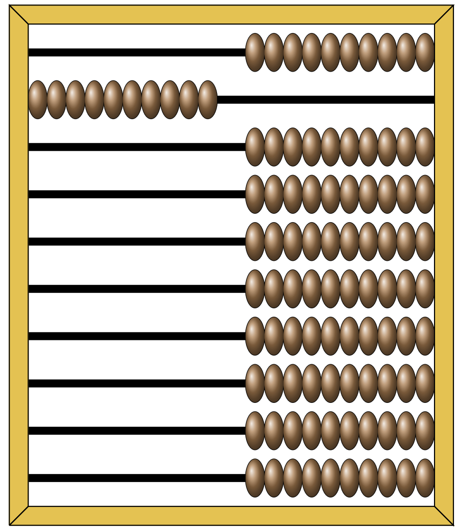
Problem 1.4 *If we were dinosaurs and had three fingers on each of our front paws, how many beads would there have been on each wire? Why?*



Problem 1.5 *Write the numbers shown on the two abaci below under the abaci.*



Problem 1.6

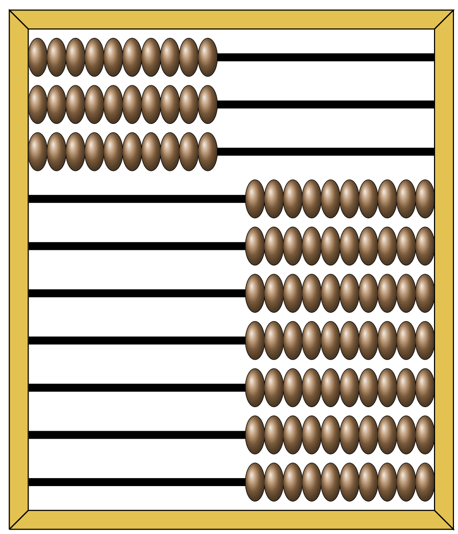


- *What number is shown on the abacus to the right of this text?*

The number is ______ .

- *What is a better way to show this number on the abacus?*

Problem 1.7



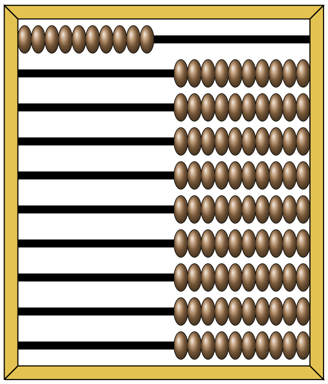
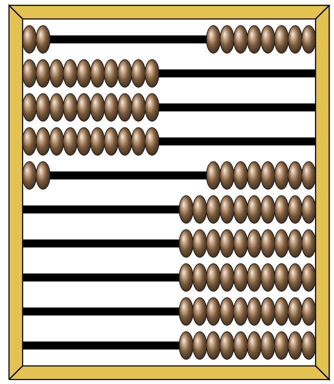
- *What number is shown on the abacus to the right of this text?*

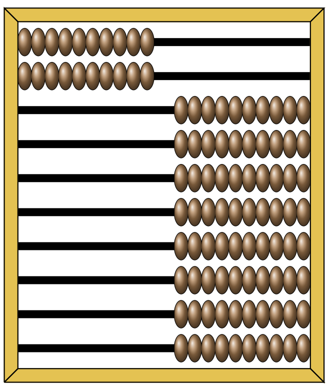
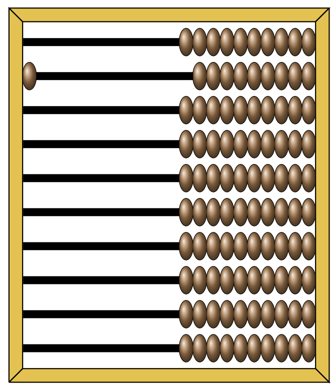
The number is ______ .

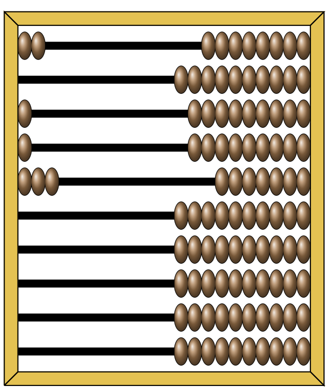
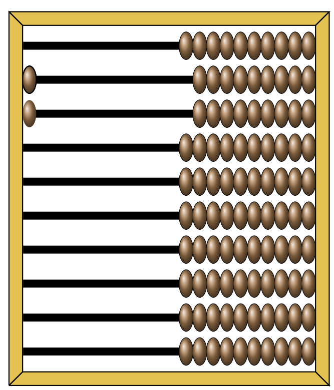
- *What is a better way to show this number on the abacus?*

Problem 1.8 *Connect an abacus on the left-hand side of the picture on the next page to the abacus showing the same number on the right-hand side of the picture. Out of the two abaci showing the same num-*

ber, circle the one representing the number in a more efficient way.

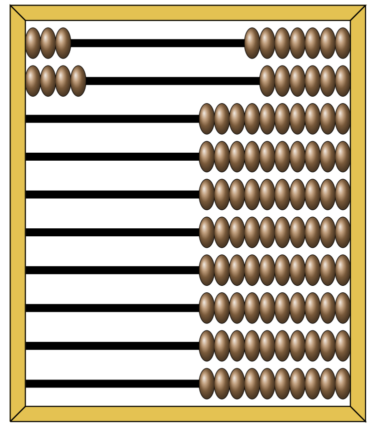



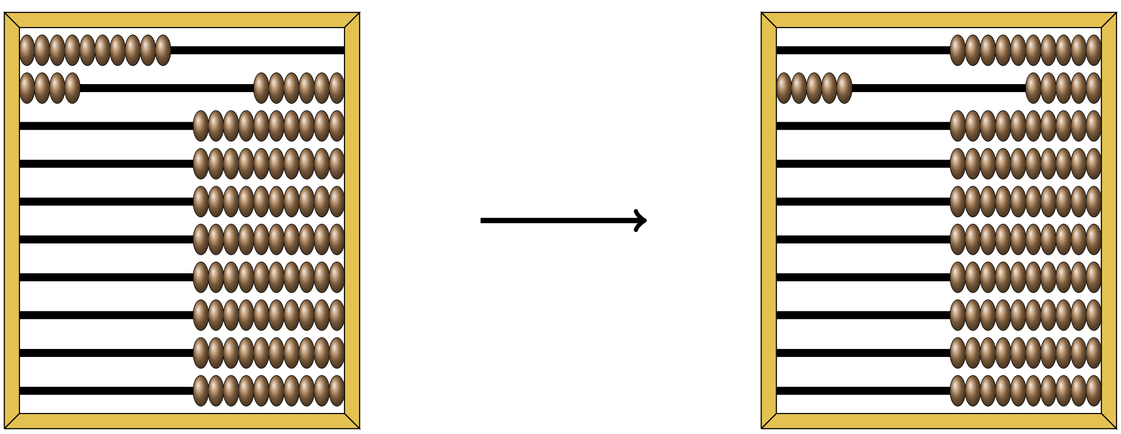
Example 1.1 *Let us use the abacus to solve the* 43 + 59 *addition problem.*

The first step is to make the number 43 *on the abacus.*



To add 59*, let us first add* 9 *and then* 50*. In other words, let us first add nine ones, then five tens. The problem is that we only have seven beads available on the first wire. Let us add these seven and bear in mind that we need to add two more.*

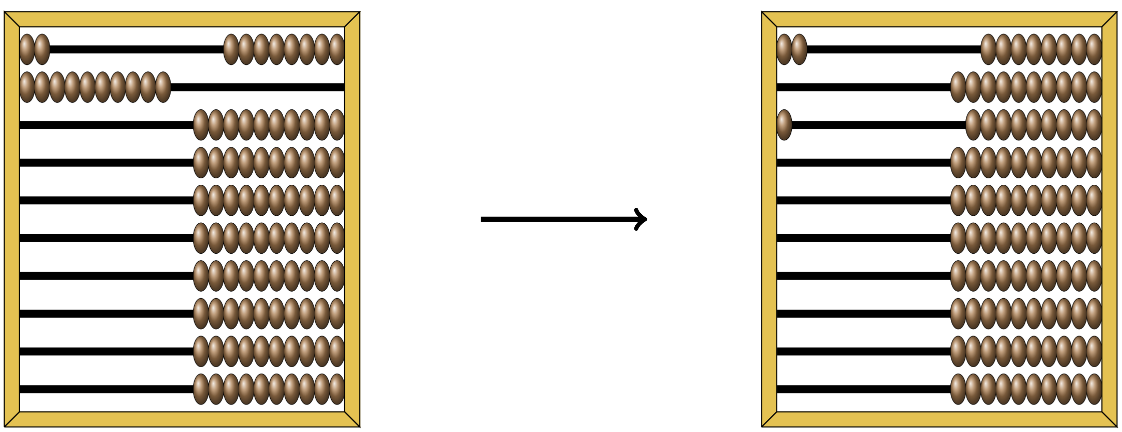
There are three beads on the first wire. When we add seven more, we get ten. But ten beads on the first wire are equivalent to one bead on the second! Please see the two abaci below.



Replacing ten beads on an upper wire with one bead on the lower wire is called <u>carrying over</u>. We just have used it to add seven ones to 43*. The result is five beads on the second wire. This corresponds to the number* 50*. Please see the right-hand side abacus on the previous page.*

Recall that we have two more ones to add because we need to add nine ones altogether. We can do it now – all the ten beads on the upper wire are available! The next step is to add five tens, that is five beads on the second wire. We already have five beads on the second wire. Adding five more makes it ten. Please see the left-hand side abacus below.

Let us carry over and replace ten beads on the second wire with one bead on the third wire.



The end result is one bead on the third wire, zero beads on the second wire, and two beads on the first wire.

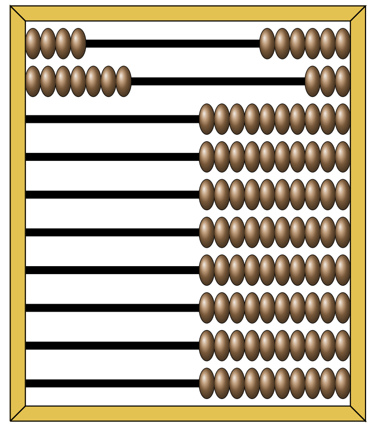
$$43 + 59 = 102$$

Problem 1.9 *Use the abacus to solve the following addition problems.*

- $99 + 3 =$
- $45 + 58 =$
- $126 + 78 =$
- $345 + 587 =$
- $1967 + 51 =$
- $2018 + 2019 =$
- $46 + 57 + 68 =$
- $103 + 2045+$
  $+ 30678 =$

Example 1.2 *Let us use the abacus to solve the* $74 - 56$ *subtraction problem.*

The first step is to make the number 74 *on the abacus.*

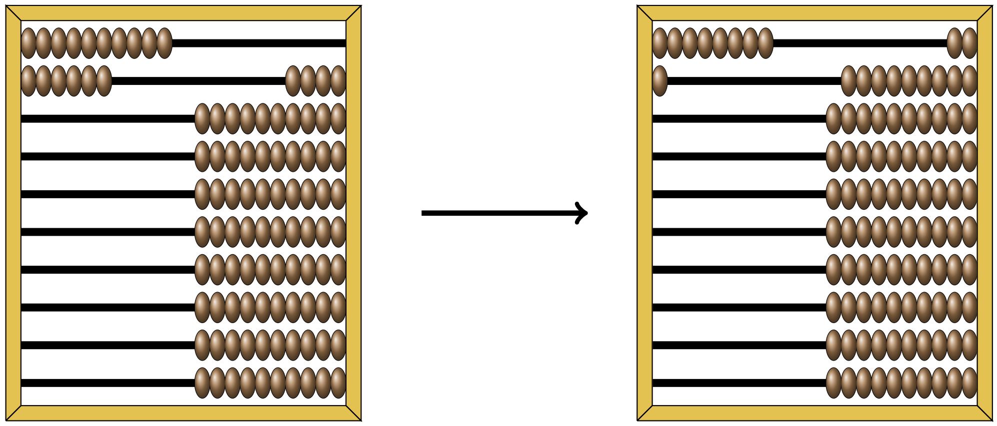


To subtract 56*, let us first subtract* 6 *and then* 50*. In other words, let us first take away six ones, then five tens. The problem is that we only have four beads available on the first wire. Let us take away these four and bear in mind that we need to take away two more.*

Since one bead on the second wire is equivalent to ten beads on the first wire, let us take away one of the seven beads on the second wire and replace it with ten beads on the first wire. Please see the left-hand side abacus below.

The operation of replacing one bead on a lower wire by ten beads on the upper wire is called <u>*borrowing*</u>*.*

Thanks to borrowing, we now have ten beads on the first wire, so we can take away the remaining two ones. We can also take away five tens out of the remaining six tens. Please see the right-hand side abacus on the next page.



The end result is one bead on the second wire and eight beads on the first wire.

$$74 - 56 = 18$$

Problem 1.10 *Use the abacus to solve the following subtraction problems.*

- $11 - 3 =$
- $45 - 26 =$
- $126 - 84 =$
- $126 - 87 =$
- $1000 - 1 =$
- $1000 - 11 =$
- $2019 - 51 =$
- $1,000,000,000 - 1 =$

Problem 1.11 *Anna makes a number using the top three wires of her abacus. Her brother Bob copies her in a funny way. First, he puts all the beads to the left. Then, he moves as many beads to the right as Anna has moved to the left. Then, he reads off his number the usual way.*

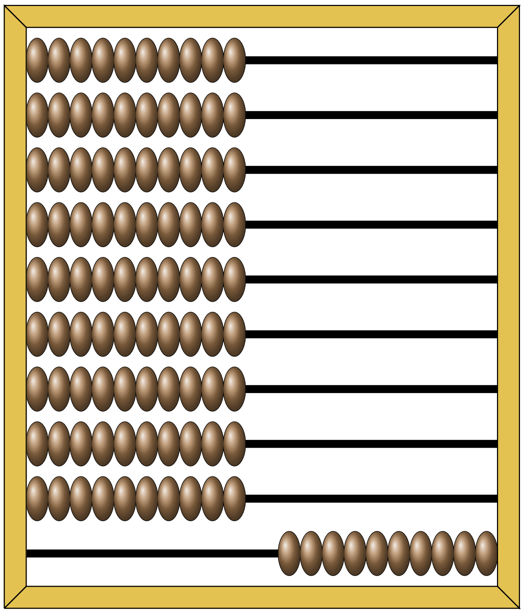
Anna and Bob add their numbers. What number do they get?

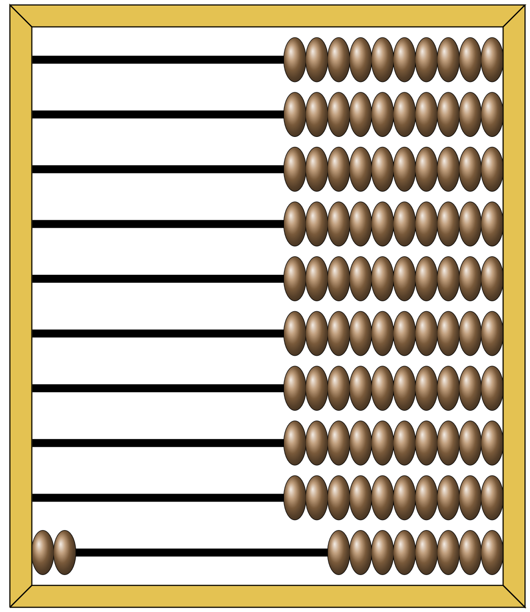
They get the number __________ .

Problem 1.12 *Cindy wants to make the greatest possible number by moving four beads on her abacus to the left. What is the greatest number she can make?*

The greatest number is __________ .

Problem 1.13 *The two abaci below show two different numbers. Which number is greater, the one on the left or the one on the right? Please circle the correct answer.*





left *right*

Problem 1.14 *Insert the total of three signs, + and −, between the digits below to make a correct statement.*

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 = 100$$

1.2 Homework

Finish solving all the problems unfinished in class.

Problem 1.15 *Use the abacus to solve the following addition problems.*

- $89 + 53 =$
- $65 + 56 =$
- $174 + 851 =$
- $999 + 888 =$
- $3972 + 6877 =$
- $10995 + 9366 =$
- $467 + 578 + 689 =$
- $1033 + 6545 + 30878 =$

Problem 1.16 *Donna wants to make the greatest possible number by moving twelve beads on her abacus to the left. What is the greatest number she can make?*

The greatest number is ____________ .

Problem 1.17 *Use the abacus to solve the following subtraction problems.*

- $21 - 7 =$
- $105 - 26 =$
- $926 - 284 =$
- $1325 - 487 =$
- $10000 - 10 =$
- $11000 - 11 =$
- $2715 - 357 =$
- $1,000,000,000$ $-111 =$

Problem 1.18 *Playing with her abacus, Emily uses the top two wires only. She moves two beads to the left to make the first number. Then the girl subtracts a one-digit number. The result requires three beads to make. Write down all the subtraction problems Emily could make.*

Problem 1.19 *On planet Octonia, the number of beads on each wire of the abacus is eight. How many fingers do the inhabitants of this planet have?*