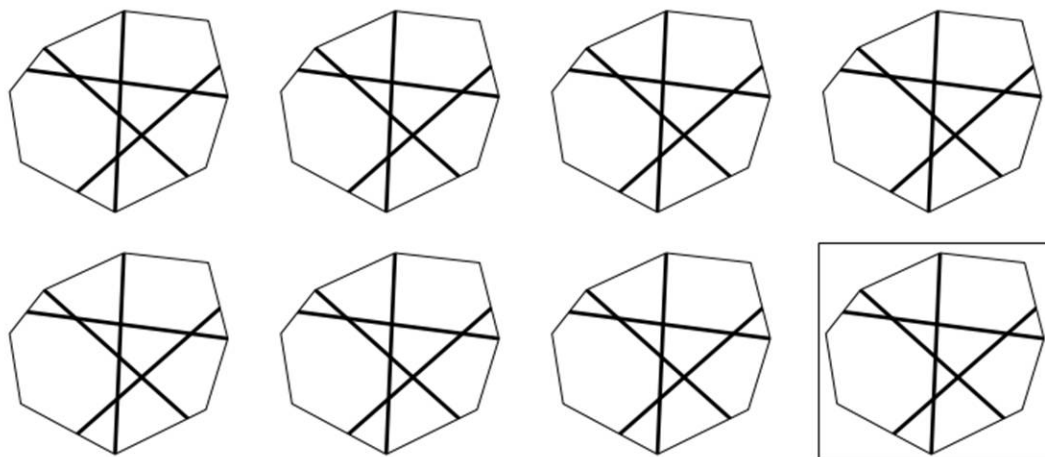


Math Circle  
Beginners Group  
October 4, 2015

**Warm-up problem**

You have 100 bags of coins. Each bag has 100 coins, but only one of these bags has special gold coins. Each gold coin weighs 1.01 ounce and all the other mediocre coins weigh 1 ounce each. You also have a weighing scale, but it is running out of battery. You can only use the scale once. How will you identify the bag with the special gold coins?

1. Four roads are crossing as shown below. Plant four trees so that the number of trees on both sides of each road is the same. Put your answer in the box, and use the other maps for experimenting.



2. A mathematician took his 5 children out for pizza.

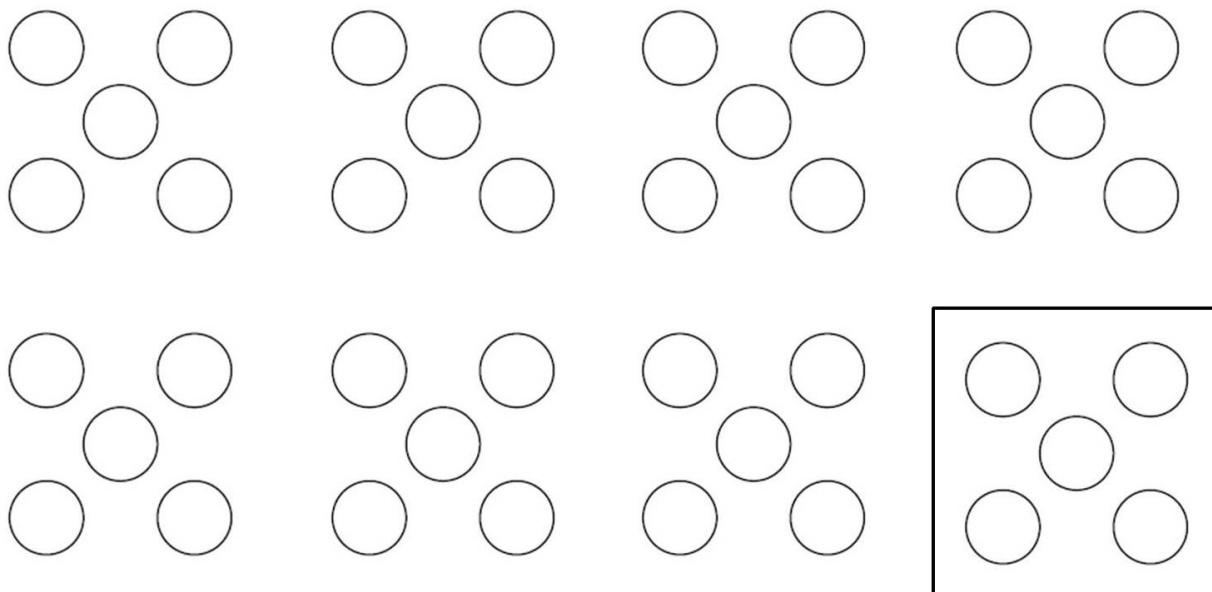
- Maria said, *“I must have tomatoes on my pizza, but no sausage, please!”*
- Daria said, *“No tomatoes for me!”*
- Nicolas continued, *“I like tomatoes, but can’t stand mushrooms”.*
- Igor said, *“I don’t like mushrooms too, but I would like sausage”.*
- Peter said, *“I would like mushrooms”.*

It is clear that you cannot order just one pizza to satisfy all tastes. Is it possible to order two pizzas so that everyone is happy?

3. Put a non-zero digit inside of each of the circles in such a way that

- the sum of the digits in the two circles in the top row is 7 times smaller than the sum of the rest of the digits; and
- the sum of the digits in the two circles on the left is 5 times smaller than the sum of the rest of the digits.

Then show that the problem has a unique solution.



(a) Let  $S$  be the sum of all the digits in the circles. What portion of  $S$  is the sum of the two digits in the top row?

(b) What portion of  $S$  is the sum of the two digits on the left?

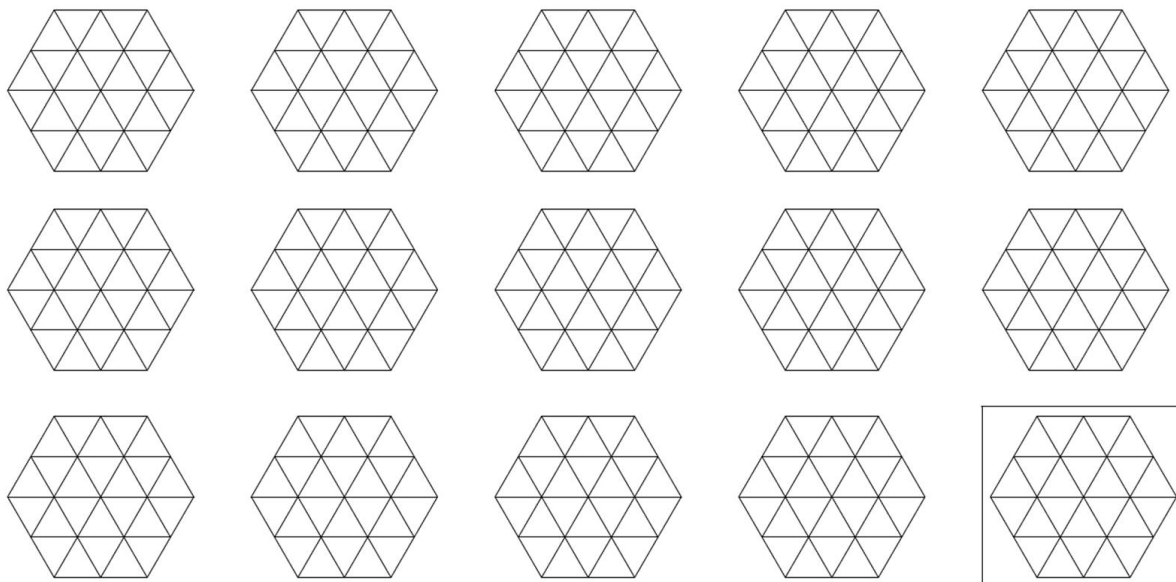
(c) Using (a) and (b), find what numbers should  $S$  be divisible by.

(d) List the possible values of  $S$ . Which of these does not work, and why?

(e) Find the digits in the circles using the value of  $S$

(f) Why does this show that there is only one solution?

4. Cut the hexagon below into 4 parts equal in shape and size. You can cut only along the grid shown on the pictures.



5. A monkey becomes happy if it eats 3 different types of fruit. You have 20 pears, 30 bananas, 40 apples and 50 mandarins. How many monkeys can you make happy?

6. George drew a rectangle on a square paper and then drew a picture inside of the rectangle. After that, he surrounded the picture by a frame which was one square wide, as shown below. It turned out that the area of the frame was equal to the area of the rectangle inside of which the picture was drawn. What are the possible sizes that George's picture could have had? List all possibilities and prove that there are no others.

