

1 How to fight a dragon

Materials for the lesson and homework: a couple of regular pencils, an eraser, a pencil sharpener, craft sticks.

Warm-up

Problem 1.1 *Can you cut a square into seven smaller squares not necessarily having the same size? If you think you can, please draw a picture in the space below. If you think that it is not possible, please explain why.*

1.1 Lesson

On a planet similar to Earth in a galaxy far far away, there live heroes and dragons. Dragons on the planet look more like hot air balloons than like big bats you see in Hollywood movies. Their bodies have spherical shape with different numbers of heads and tails growing out of them. We write the numbers of heads and tails a dragon has as (H, T) , first the number of heads then the number of tails. For example, a dragon having five heads and eight tails is represented by the pair of numbers $(5, 8)$. We call such a pair a *vector*. The order of numbers in a vector is very important. For example, the vector $(8, 5)$ represents a dragon with

eight heads and five tails, very different from a dragon having five heads and eight tails.

Problem 1.2

- *How many heads and tails has a dragon represented by the vector $(7, 2)$?*

The dragon has _____ heads and _____ tails.

- *Write down a vector representing a dragon with one head and four tails.*

$(\text{_____}, \text{_____})$

Occasionally, a hero and a dragon get into a fight. The following are the rules.

1. With one blow of a sword, a hero can cut off either one or two heads, or either one or two tails.
2. If one head is cut off, two heads instantly grow instead.
3. If two heads are cut off, nothing grows back.
4. If one tail is cut off, two tails instantly grow back.
5. If two tails are cut off, one head grows instead right away.

For example, if a dragon originally had five heads and eight tails and one of its heads was cut off, then the dragon ends up having six heads and

eight tails. Indeed, the dragon loses one head out of five, but, according to rule 2, two more heads instantly grow instead.

$$5 - 1 + 2 = 6$$

The number of the tails does not change, so the event is described by the following diagram.

$$(5, 8) \xrightarrow{1H} (6, 8)$$

If a dragon loses two heads, we put $2H$ above the arrow. The notation for one lost tail is $1T$. The notation for two lost tails is $2T$.

To defeat a dragon, a hero has to cut off all of its heads and tails. Since a defeated dragon has neither heads nor tails, it is represented by the state vector $(0, 0)$. A defeated dragon actually does not die. It just drifts headlessly away.

Problem 1.3 Use rules 1-5 to make diagrams corresponding to each of the fight events below. As an example, the first event is worked out for you.

- A dragon originally had two heads and two tails. One of its heads was cut off.

$$(2, 2) \xrightarrow{1H} (3, 2)$$

- A dragon originally had three heads and one tail. Two of its heads were cut off.

$$\left(\underline{\quad}, \underline{\quad} \right) \longrightarrow \left(\underline{\quad}, \underline{\quad} \right)$$

- A dragon originally had six heads and seven tails. One of its tails was cut off.

$$\left(\underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right) \longrightarrow \left(\underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right)$$

- A dragon originally had one heads and two tails. The tails were cut off.

$$\left(\underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right) \longrightarrow \left(\underline{\hspace{1cm}}, \underline{\hspace{1cm}} \right)$$

Problem 1.4 *The original and final state of a dragon are described by pairs of vectors connected by arrows below. Figure out what happened to each dragon in a fight and write down notations of the events in the boxes above the arrows.*

- | | |
|-----------------------------------------|-----------------------------------------|
| • $(5, 6) \xrightarrow{\square} (5, 7)$ | • $(3, 3) \xrightarrow{\square} (4, 3)$ |
| • $(2, 5) \xrightarrow{\square} (0, 5)$ | • $(2, 3) \xrightarrow{\square} (3, 1)$ |

Problem 1.5 *Given the final state and the name of the event, find the original state for each of the events below.*

- $(\text{---}, \text{---}) \xrightarrow{2H} (4, 4)$

- $(\text{---}, \text{---}) \xrightarrow{1T} (2, 3)$

- $(\text{---}, \text{---}) \xrightarrow{1H} (3, 7)$

- $(\text{---}, \text{---}) \xrightarrow{2T} (7, 8)$

This way, you can write down an entire fight! For example, here is a way to defeat a $(3, 0)$ dragon:

$$(3, 0) \xrightarrow{1H} (4, 0) \xrightarrow{2H} (2, 0) \xrightarrow{2H} (0, 0).$$

Problem 1.6 *Come up with a plan to defeat the following dragons as quickly as possible.*

- $(1, 0) \longrightarrow$

- $(2, 1) \longrightarrow$

- $(4, 1) \longrightarrow$

- $(3, 2) \longrightarrow$

- $(1, 1) \longrightarrow$

- $(7, 8) \longrightarrow$

Problem 1.7

- *One of the biggest dragons on the planet currently has 2,019 heads and no tails. How would you fight this dragon?*

- *Another big dragon has one head and 2018 tails. How would you defeat this one?*

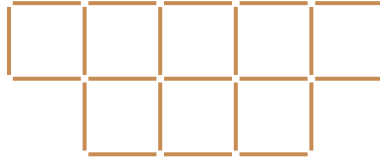
Problem 1.8

- *How would you defeat a dragon with an even number of heads and no tails?*

- *How would you defeat a dragon with an odd number of heads and no tails?*

Problem 1.9 *How would you fight a dragon with one head and any number of tails? Hint: consider the cases when the number of tails is even and odd separately.*

Problem 1.10 *› The picture below is made of craft sticks. Remove three of them so that the remaining sticks form five squares of equal size.*



1.2 Homework

Challenge your friends and family members to defeat a dragon with the number of heads and tails of your choosing.

Problem 1.11 *There are three apples in a basket. Find a way to split the apples between three persons so that each person gets an apple and one apple remains in the basket.*