Fibonacci Numbers

November 7, 2010

Fibonacci's Task: Figure out how many pairs of rabbits there will be at the end of one year, following rules.

Rules:
1. Start with a pair of new rabbits, born in December.
2. These adult rabbits start having children when they are two months old.
3. Each pair of rabbits always has one new pair of rabbits (children) each month, starting when they are two months old.
4. Rabbits never die.

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1. A Rabbit Family consists of a pair of parents and only the children of those parents. If Peter and Penny Rabbit are the original pair of rabbits, born in December ("Month 0"), fill in the table below to count the total number of pairs of rabbits in their family in the 12 months of the following year. Remember, Peter and Penny begin having children when they are 2 months old, in Month 2, and they only have *one* new pair of children per month.

Month	0	1	2	3	4	5	6	7	8	9	10	11	12
# of pairs in Family	1	1	2										

Now, consider the fact that each pair of children also start having children of their own:

* In the month that they are born in, the rabbits are called *newborn*;

* In the next month, they are *teenagers* and do not have children yet;

* In the month after that, rabbits become *adults* and start having children (one pair each month). Because rabbits never die, once rabbits become adults, they remain adults forever.

2. If a pair of rabbits is born in March, when will their first pair of children be born?

3. Suppose that we start with 1 pair of newborn rabbits in Month 0 (December).

    (a) How many pairs of each type of rabbits (newborns, teenagers and adults) are there in Month 1?

    (b) What about Month 2?

4. In any month, only adult rabbits can have children. Each pair of adult rabbits produces only one pair of newborn rabbits per month.

    (a) If there are 6 pairs of adult rabbits in September, how many pairs of rabbits will be born that month?

    (b) Can you state, in general, how the number of pairs of adults and the number of pairs of newborn rabbits in a given month are related? (That is, if A pairs of rabbits are adults in a month, how many pairs of newborn rabbits will there be that same month?)

2

5. Suppose that in March there are 8 pairs of adult rabbits and 3 pairs of teenage rabbits.

   (a) Do you have enough information to figure out how many pairs of rabbits will be adults in April? If so, what is the answer?

   (b) Can you also find out how many pairs of newborn rabbits will be born in April?

6. In general, how is the numbers of pairs of teenage rabbits and pairs of adult rabbits in one month related to the number of adult rabbits in the next month? (That is, suppose you have T pairs of teenage rabbits and A pairs of adult rabbits in April. What is the number of pairs of adult rabbits in May?)

7. How is the number of pairs of newborn rabbits in one month related to the number of pairs of teenage rabbits in the next month? (That is, if there are N pairs of newborn rabbits born in August, how many teenage rabbits will there be in September?)

8. Use the general ideas you came up with in the previous problems to fill in the table below:

Month		0	1	2	3	4	5	6	7	8	9	10	11	12
Old pairs of rabbits	Adult pairs	0	0	1	1									
	Teenage pairs	0	1	0	1									
New pairs	Newborns	1	0	1	1									
Total:		1	1	2	3									

9. Now, instead of considering the "teenage" and "adult" rabbits individually, we'll call any rabbits which are not newborns the "Old" rabbits. Rewrite the table above again, but this time, add the number of pairs of "teenagers" and "adults" together to get the number "Old" pairs of rabbits in each month. ("New" pairs of rabbits are the newborns).

Month	0	1	2	3	4	5	6	7	8	9	10	11	12
"Old" pairs	0	1	1	2									
"New" pairs	1	0	1	1									
Total pairs:	1	1	2	3									

You can use the space below for any calculations:

10. How is the number of "old" pairs of rabbits in a month related to the *total* number of pairs of rabbits the previous month?

11. How is the number of "new" pairs of rabbits in a month related to the *total* number of pairs of rabbits in the month that comes 2 months before the current month?

12. Notice that there is a pattern to these numbers. How is the total number of pairs each month related to the previous numbers of pairs of rabbits in the previous two months?

13. If there is a total of 4 pairs of rabbits (old and new) in February, and 8 pairs of rabbits in March, what is the total number of pairs of rabbits that you have in April?

14. Could you say how many pairs of rabbits there will be in the 13th month (the following January)?

15. The sequence of numbers expressing the total number of pairs of rabbits in each month (starting with the 0th month) looks as follows:

$$1, \ 1 \ , 2 \ , 3 \ , 5 \ , 8 \ , 13 \ , 21 \ , 34 \ , 55, \ \ldots.$$

It is called the *Fibonacci sequence* and plays an important role in many areas of mathematics.

   (a) Can you explain how a number in the sequence above is related to the two numbers just before it?

   (b) Suppose that someone wrote out this sequence up to a certain point and asked you to come up with the next number in the sequence. What do you need to do?

16. END CHALLENGE: Create a pattern similar to above, but this time, think about starting with 2 pairs of rabbits born in Month 0. (We assume that the "rules" stated above are still true). Write out the first few numbers of the sequence. How many pairs of rabbits would there be at the end of one year this time?

17. ONE MORE CHALLENGE: Starting the sequence with the numbers 1 and 3, use the rule you have found in Problem 15a to write down the first 10 terms of the sequence.