Knights, Knaves, and Knormals

Who should you trust
if you can't trust anyone?
Many problems taken from
'The Lady or the Tiger' by Raymond Smullyan.

Math Circle

April 21, 2019

Today we are going to do lots of logic puzzles! But before we get into problems involving knights knaves and knormals (the k is silent) let's warm up with some logic puzzles.

1. These first puzzles are warm up.

   (a) You have a drawer where you put all of your socks. In that drawer there are 12 blue, and 12 red pairs of socks (so 48 total). A sock fits just as well on the left foot as right foot. If your room is dark, and you can't tell what color the socks are, how many socks must you take in order to guarantee that you select two socks of the same color?

   (b) What about if you have fancy ergonomic socks which come in left and right foot versions? How many do you have to pick in order to guarantee that you have at least one pair of socks of the same color?

(c) A new twist on problem 1.a: Suppose some blue socks and the same number of red socks are in a drawer. Suppose that it turns out that the minimum number of socks that I must pick in order to be sure of getting at least one pair of the same color is the same as the minimum number I must pick in order to be sure of getting at least two socks of different colors. How many socks are in the drawer?

2. Ok, let's get onto the problems involving k**N**ights, k**N**aves and k**N**ormals (**NNN**). These kinds of logical puzzles have been popular for a long time, so you might have heard some before. The idea is this:

   You go to visit a far off island of Rayland. Everyone on Rayland (other than you) is either a knight or a knave (we'll talk about knormals later). Knights will always tell the truth, while knaves will always lie.

   (a) You come across three folks sitting in a garden. Let's call them A, B and C. You ask A if they are a knight. A answers, but you can't quite make out what they said. You ask B "What did A say?" B replies "A said that she was a knave." At this moment C says "Don't believe B! He is lying!" The question is, what are B and C?

(b) You walk on a bit further, and you see two little kids, A and B, climbing a tree. You ask them who they are and A responds "At least one of us is a knave." What are A and B?

(c) Suppose instead that A had said "Either I am a knave or B is a knight." What are A and B?

(d) Suppose instead that A had said "Either I am a knave or else two plus two equals fish." What could you conclude?

(e) You continue walking across the Rayland, and come across another three people, A B and C each of whom is either a knight or a knave. A and B make the following statements.

A: All of us are knaves
B: Exactly one of us is a knight

What are A, B and C?

(f) Suppose instead that A and B had said the following

A: All of us are knaves.
B: Exactly one of is is a knave

What can you conclude?

(g) Suppose that A had said ”I am a knave, but B isn’t”

What are A and B?

(h) We again have three inhabitants, A, B and C, each of whom is either a knight or a knave. Two people are said to be of the same type if they are both knights or both knaves. A and B make the following statements:

A: B is a knave.
B: A and B are of the same type.

What can you conclude?

(i) Again three people A, B and C. A says B and C are of the same type.” Someone then asks C, ”Are A and B of the same type?” What does C answer?

(j) You walk a bit further still and see two inhabitants of Rayland laying under a tree. You ask one of them, "Are either of you a knight?" She responded, and you knew the answer to your question.

Was the person that you asked the question to a knight or a knave? How about the other person?

3. You decide that you have seen quite enough of Rayland, and decide to leave to Smullville. Smullville is also inhabited by knights and knaves, but it also has a third kind of inhabitant, knormals. Knormals sometimes tell the truth, and other times lie. It is here where our next few puzzles take place

   (a) We are given three people, A, B and C, one of whom is a knight, one of whom is a knave, and the last of whom is a knormal. They make the following statements:

   A: I am the knormal.
   B: That isn't true!
   C: I am not knormal.

   What can you conclude?

(b) Two people, A and B say the following:

A: B is a knight.
B: A is not a knight.

Prove that at least one of them is telling the truth, but is not a knight.

(c) This time A and B say the following:

A: B is a knight.
B: A is a knave.

What can you conclude?

(d) On the island of Mullville, there is a caste system. knights belong to the highest caste, normals below, and knaves the lowest caste. Two inhabitants make the following statements:

A: I am a lower rank than B.
B: That's not true!

What can you conclude?

(e) Given three people A, B and C, one of whom is a knight, one a knave, and the last a knormal. A and B say:

A: B is of higher rank than C.
B: C is of higher rank than A.

Then C is asked, "Who has the higher rank, A or B?" What does C answer?

4. The last stop on our journey is the island of Merrillia. On the island of Merrillia, a knight can only marry a knave, a knave could only marry a knight, and so a knormal could marry a knromal.

(a) The first people that we meet on the island are Mr. and Mrs. A. They make the following statements.

Mr. A: My wife is not a knormal.
Mrs. A: My husband is not knormal.

What are Mr. and Mrs. A?

(b) Suppose instead that they had said:

Mr. A: My wife is a knormal.
Mrs. A: My husband is knormal.

Would the answer have been different?

(c) The final people that you meet on this island are a pair of couples, Mr and Mrs A, and Mr and Mrs B. They say the following:
Mr. A: Mr. B is a knight.
Mrs. A: That's right, Mr. B is a knight.
Mrs. B: Indeed! My husband is a knight.

What can you conclude?

5. Finally here are some bonus logic puzzles that you can work through that don't have to do with knights, knaves or knormals. They instead have to do with a hat-obsessed wizard.

   (a) A wizard captures three prisoners and puts a hat on each of them (they can't see their own hats). He sits them in a line facing forward, so that prisoner 1 can see prisoners 2 and 3 (and their hats), prisoner 2 can see prisoner 3, and prisoner 3 can't see anyone. The wizard tells the prisoners that he has 5 hats total, three white hats and two black hats. He then puts three hats on the three prisoners. He then asks the prisoners in turn what color their hat's are.

   Prisoner 1 says: "I don't know what color my hat is."
   Prisoner 2 says: "I don't know what color my hat is."
   Prisoner 3 says: "Oh! Then I know what color my hat is!"

   How did prisoner 3 know what color his hat was? What color was it?

   (b) The wizard captures 100 people and throws them in prison overnight. He tells them that tomorrow he will put hats on them and have them sit down in the same way as before (so prisoner 1 can see 2 - 99, prisoner 2 can see 3 - 99, etc...). He then says that he will start with prisoner 1 and ask them what color their hat is. He cast a spell on the prisoners so that they can only say "White" or "Black," and nothing else. If they guess their own hat color right he will let them go, otherwise he will turn them into a pig.

   He also tells his prisoners that he will randomize the order in which they sit, and he has 100 white and 100 black hats total. Overnight the prisoners are allowed to scheme with each other to make sure that as few of them as possible end up as pigs. What should the prisoners do?

(c) The wizard captures three more prisoners. He has them sit in a circle, so that each prisoner can see the other two prisoner's hats. This time the hats come in one of three colors: White, Black, and Red. The wizard counts to three, waves his magic wand and each prisoner must say either "White," "Black," or "Red." If *any* of the prisoners guess their hat color correctly then they all go free. Otherwise they all turn into pigs. What strategy should the prisoners use to make sure that they are as likely to escape as pig-less as possible?