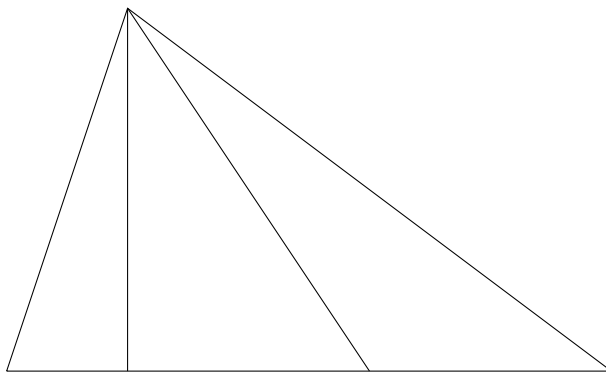# 14   Odd and even numbers

Materials for the lesson and homework: a couple of regular pencils, an eraser, a pencil sharpener.

Warm-up

Problem 14.1 *How many triangles are there on the picture below?*

There are _____ *triangles.*

Lesson

Recall that to *double* a number is to add the number to itself. For example, the double of 3 is $6 = 3 + 3$.

Problem 14.2

- *What is the double of the number* 64*?*

The double is the number _____ .

- *What is the double of the number* 2019*? If needed, use the abacus to figure out.*

The double is the number _____ .

- *Alice doubled a number and got* 64*. What was the original number?*

The original number was _____ .

- *Cindy doubled a number and got* 250*. What was the original number?*

The original number was _____ .

An integer is called *even*, if it is a double of another integer. An integer is called *odd* otherwise. For example, the number 10 is even because $10 = 5 + 5$. The number 11 is odd for the following reason: $5 + 5 = 10$ and $6 + 6 = 12$. Note that $10 < 11 < 12$, but there is no integer between 5 and 6. Thus, there is no integer one can double to get 11. So, 11 is not even. Therefore, it is odd.
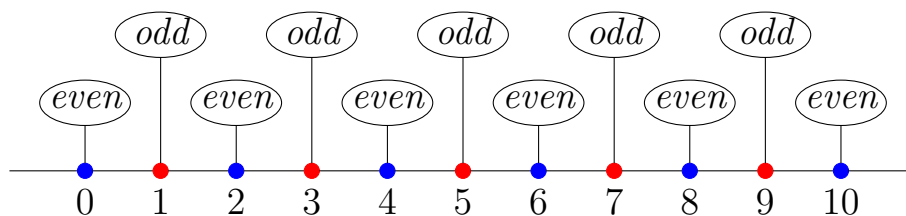
Problem 14.3 *Is the number* 0 *odd or even? Circle the correct answer.*

Odd   Even

Explain your choice.

Note that odd and even numbers alternate on the number line. Can you explain why?

Problem 14.4

- *Is the number 22 odd or even? Circle the correct answer.*

$$\textit{Odd} \qquad\qquad \textit{Even}$$

Explain your choice.

- *Is the number 21 odd or even? Circle the correct answer.*

$$\textit{Odd} \qquad\qquad \textit{Even}$$

Explain your choice.

Problem 14.5

- *Write down the first five basic binary numbers in decimal notations.*

$$\text{———} \;,\; \text{———} \;,\; \text{———} \;,\; \text{———} \;,\; \text{———}$$

- *Circle odd basic binary numbers. Do not circle the even ones. Explain your choice.*

Problem 14.6 *Explain the following formulas.*

- **even + 1 = odd**

- **odd + 1 = even**

   Two numbers are called *consecutive*, if one of them follows the other as we count. For example, the numbers 9 and 10 are consecutive.

Problem 14.7 *The sum of two consecutive numbers equals* 15. *What are the numbers?*

The numbers are _____ *and* _____ .

Problem 14.8 *Ben has a book open in front of him. The sum of the numbers of the pages he is looking at is* 29. *What are the numbers of the pages Ben is looking at?*

The numbers are _____ *and* _____ .

Problem 14.9 *Is the sum of two consecutive numbers always odd, always even, or can it be either? Circle the correct answer.*

Odd   *Even*   *Either*

Explain your choice.

Problem 14.10 *Binary Ben has a magazine open in front of him. The sum of the binary numbers of the pages he is looking at is B* 101. *What are the binary numbers of the pages Binary Ben is looking at?*

The numbers are B _____ *and B* _____ .

Problem 14.11 *For the binary numbers below, decide whether each of them is odd or even, circle the correct answer and explain your choice.*

- B 10 :   *Odd*   *Even*

- B 101 :   *Odd*   *Even*

- B 110 :   *Odd*   *Even*

- B 1011 :   *Odd*   *Even*

Problem 14.12

- *A binary number ends with 1. Is it odd, even, or can it be either? Circle the correct answer.*

    Odd   *Even*   *Either*

Explain your choice.

- *A binary number ends with 0. Is it odd, even, or can it be either? Circle the correct answer.*

    Odd   *Even*   *Either*

Explain your choice.

Problem 14.13 *Explain the following formulas.*

- **even + even = even**

- **odd + even = odd**

- **odd + odd = even**

Problem 14.14 *Snowwhite wrote the number* 20 *on a piece of paper and gave it to the seven dwarfs. Each of the dwarfs either added or subtracted one from the number he got, marked out the old number and wrote down the new one.*

- *Could they get* 21 *as the final number? If you think they could, show how. If you think they couldn't, explain why.*

- *How about* 23*?*

- *How about* 22*?*

- *How about* 18*?*

- *How about* 17*?*

- *List all the numbers the dwarfs could get.*

Question 14.1 *What is a natural number?*

Problem 14.15

- *There are five natural numbers on a circle. Show that you can always find a pair of neighboring numbers with an even sum.*

- *There are 2019 natural numbers on a circle. Show that you can always find a pair of neighboring numbers with an even sum.*

Try to solve the following problem only if you know multiplication. Skip it otherwise.

Problem 14.16 ❭ *Lines from the top vertex to the base of a triangle are numbered from 1 to 100 as shown on the picture below. How many triangles are there on the picture?*

1 2 3...   ... 98 99 100
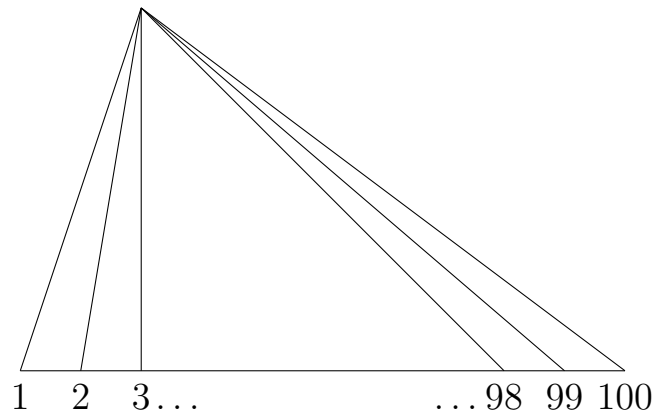
Homework

Finish all the problems from class.

Problem 14.17

- *Is the number* 102 *odd or even? Circle the correct answer.*

$$Odd \qquad\qquad Even$$

Explain your choice.

- *Is the number* 103 *odd or even? Circle the correct answer.*

$$Odd \qquad\qquad Even$$

Explain your choice.

Question 14.2 *What numbers do we call consecutive?*

Problem 14.18

- *The sum of two consecutive numbers equals* 43. *What are the numbers?*

The numbers are _____ *and* _____ .

- *The sum of three consecutive numbers equals* 63. *What are the numbers?*

The numbers are _____ , _____ , *and* _____ .

Problem 14.19 *Binary Bob has a book open in front of him. The sum of the binary numbers of the pages he is looking at is B 1101. What are the binary numbers of the pages Binary Bob is looking at?*

The numbers are B _____ *and B* _____ *.*

Problem 14.20 *For the binary numbers below, decide whether each of them is odd or even without converting to decimals. Circle the correct answer and explain your choice to your parents.*

- *B* 111010 :   *Odd*   *Even*

- *B* 1011001 :   *Odd*   *Even*

- *B* 10000010 :   *Odd*   *Even*

- *B* 11111011 :   *Odd*   *Even*

Problem 14.21 *Is the sum of the numbers*

$$1 + 2 + 3 + \ldots + 98 + 99 + 100$$

odd or even? Circle the correct answer.

Odd   *Even*

Explain your choice.

Problem 14.22 *There are five numbers such that the sum of any three of them is even. Show that all the numbers are even.*

Problem 14.23 *A teacher has organized a library in her classroom. Currently, there are 40 books on the library's shelves. There are 23 students using the library. Within a week, every student either checks out a book or returns one to the library. Could there be 24 books on the library shelves at the end of the week? Circle the correct answer.*

$$Yes \qquad\qquad No$$

Explain your choice.

References

[1] O. Gleizer, O. Radko, *Breaking Numbers into Parts*, Second Edition, Part 1

[2] O. Gleizer, O. Radko, *Breaking Numbers into Parts*, Second Edition, Part 2