

Greedy Is Good

1. Different aliens use different sets of coins. Earthling coins have denominations 1, 5, 10, 25. Shmearthling coins are 4, 10, 25. The Zolgons have coins in denominations of 2, 16, and 20. The Beryllians use coins with values 3, 5, 11, 40. Gluglaxian coins have values 3, 7, 12, 24. These values are summarized in the table below.

Species	Denominations
Earthling	1, 5, 10, 25
Shmearthling	4, 10, 25
Zolgon	2, 16, 20
Beryllian	3, 5, 11, 40
Gluglaxian	3, 7, 12, 24

For each planet, decide if it's possible to make change, and if so the fewest number of coins required, for the following amounts of money:

(a) 14?

(b) 50?

(c) 41?

(d) 76?

The *Greedy Algorithm for Making Change* is a rule describing how to make change for any number, given any particular coin denominations. The rule is this:

Continually pick the largest denomination possible until you succeed in making change or you can't pick any more coins.

Sometimes the algorithm stops because you can't pick any more coins—in other words, the amount remaining to make change for is smaller than the smallest denomination.

2. If the greedy algorithm fails to make change for a certain amount of money using a certain set of coins, does this mean it's not possible to make change for that number using that set of coins?

3. For which amounts of money and coin-sets in the previous problem does the algorithm give a way to make change? For which ones does it give the *best possible* way to make change (makes change if possible, and uses the fewest coins)?

4. Are there any planets where the greedy algorithm always gives the best possible change? What is it about those planets' coin-sets that allows the algorithm to work well?

5. On Planet Kulu, the coins have values 3 and 5. What is the largest amount of money you can't make change for? Why?

The nation of Primland wants to construct a modern highway system between its cities. Since the highways are expensive, the government only wants to build enough roads so that there is at least one path between any two of the cities, possibly going through many other cities on the way. The highway engineers have made a diagram showing, for several pairs of cities, the cost (in millions of Primlandian dollars) of building a highway between that pair of cities. (If two cities are not on the diagram, it's impossible and/or prohibitively expensive to connect them directly.)

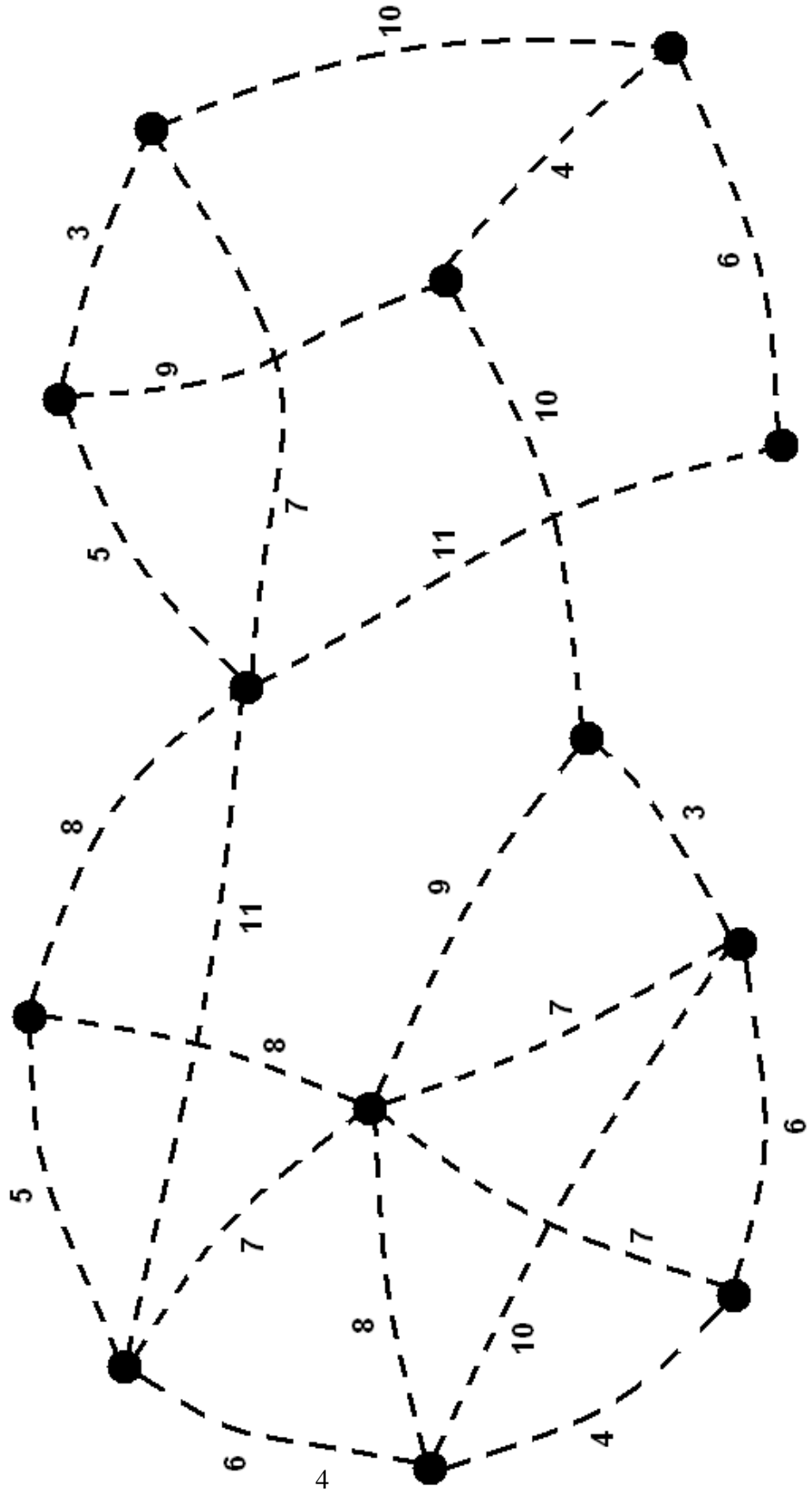
6. Design a system of highways which meets Primland's needs, at the lowest possible cost.

7. The nearby country of Kruskalia has decided to follow Primland's example and construct a new highway system as well. Its engineers have drawn up a map of possible connections for the cities of Kruskalia. Your goal is the same as before: Design a highway system such that any pair of cities is connected by highways, while minimizing the overall construction cost.

8. The League of Mathematical Nations has been greatly impressed by your work in highway system design. They've asked you to train a class of engineers who will then go design highway systems for other nations.

Try to describe a (greedy?) algorithm which will help the engineers design highway systems for new maps.

Primland



Kruskalia

