

IFF and Parity

Preston Carroll

10 February 2018

1. Difference of two numbers is even if and only if the sum of the two numbers is even.
2. Show that the number $ab(a+b)$ is even for all a and b .
3. You have 2019 numbers written on the board. Show that you erase one number so that the sum of the remaining numbers is even.

4. A product of 26 whole numbers is equal to 1. Show that their sum can not be 0.

5. There are 237 of red and blue dots on a circle.
show that there are two dots of the same color next to each other; show that there are two dots of the same color with two other dots in between;

6. Someone placed n real numbers around a circle. The sum of any two neighbors is odd. Show that the total number of numbers is even.

7. Complete the sentences:

* A product of several number is odd if and only if . . .

* (same about even)

8. a) Find 3 numbers so that their sum and product are both odd. b) same for 4 numbers
9. There are 19 students in the math circle. During the spring break, each of them wrote either 2 or 4 letters to some other math circle students. After the break, it turned out that every student received exactly 3 letters. Show that some letters got lost in the mail.
10. 101 students are taking Math Kangaroo. We know that if you select any 100 people out of these 101, there is a person among this group who knows all of the rest of the people from these 100 people. Show that there is a person who knows everyone else.