

Homework 6: Invariants and Geometric Constructions

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1 Reading

Solution 1 (L5.1).

Consider the parity of the number of minuses. When we change two pluses or two minuses to a plus, it does not change. When we change a minus and a plus to a minus, it does not change either. Thus it is an invariant, and since there is an odd number of minuses initially, there must be an odd number of minuses in the final state. But at the end there is only one sign left, and so it must be a minus.

Solution 2 (H5.1).

Consider the sum of all the numbers on the board. Every time a and b are replaced by $a + b - 1$, it decreases by 1. Thus after applying the operation 19 times, it decreases by 19 and is equal to $10 \cdot 21 - 19 = 191$. But after 19 operations there is only one number left, so it must be exactly 191.

2 Homework

Problem 1.

Given a line ℓ on the plane, construct line m such that the angle between ℓ and m is 45° .

Problem 2.

There are N numbers written on the board, each number is equal to 1 or -1 . One step consists of picking several (or one) consecutive numbers and changing their signs. What is the minimum number of steps that is sufficient to make all numbers equal to 1, no matter the starting configuration?