

Parity and Proof writing Part I

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1 Parity

1. Three hockey pucks, one red, one green, and one blue, lie on a playing field. A hockey player hits one of them in a way such that it passes between the other two. He does this 25 times. Can he return the three pucks to their starting points?
2. The product of 22 integers is equal to 1. Show that their sum cannot be zero.
3. The number 1,2,3,..., 1984, 1985 are written on a blackboard. We decide to erase from the blackboard any two numbers, and replace them with their positive difference. After this is done several times, a single number remains on the blackboard. Can this number equal 0?
4. Of 101 coins, 50 are counterfeit, and differ from the genuine coins in weight by 1 gram. Peter has a scale in the form of a balance which shows the difference in weight between the objects placed in each pan. He chooses one coin, and wants to find out in one weighing whether it is counterfeit. Can he do this?

2 Proof writing

When writing a proof, there are several techniques to approach the problem with. One of them is known as "If and Only If". In order to show this, we need to not only show "If A, then B." but also "If B, then A".

1. Take for example the statement :
A counting number is odd IF AND ONLY IF its square is odd.

What two statements do you need to prove?

2. Now let's prove those two statements:

3. Now sometimes we don't necessarily want to prove something to be true, but rather that a statement is FALSE. A common method to do this is to provide a counterexample.

Example 1: For all real numbers, $x > 0, x^3 > x^2$

First off, do you think this statement is true? When working on proving a statement, it is generally a good idea to see if you can "break" the statement. If you can find an example where you satisfy the conditions, but the conclusion fails, then you have found exactly what we are looking for.

4. Consider the following other examples and see if you can find counterexamples:

The reciprocal of a real number $x \geq 1$ is a number y such that $0 < y < 1$.

The number $2^n + 1$ is prime for all counting numbers n .

The sum of any five consecutive integers is divisible by 5.