Divisibility Tests

Problem 1) Work out divisibility tests for (powers of) 5.

Problem 2) Work out the divisibility test for 9.

Problem 3) Given a collection of distinct primes, $p_1, p_2, \ldots, p_s$ Given some integer, m, we know that

$$(p_1 p_2 \cdots p_s) \mid m \text{ if and only if } p_i \mid m \text{ for every } i.$$

With this in mind, construct divisibility tests for, say, $6, 12, 18, 30$.

Problem 4) This problem is about testing for divisibility by 7. Here's the rule you're going to prove:

Beginning with some integer x, remove the final digit (the unit entry), then subtract two times the removed digit from x. The new number is divisible by 7 if and only if the original number, x, is.

Problem 5) Using the fact that

$$1001 = 7 \times 11 \times 13 \equiv -1 \mod 1000,$$

deduce a rule to detect when a number is divisible by the above three primes.

Problem 6) Finally, let's get back to our initial problem: if

$$x = 4,599,692,453,673,240,$$

Is x a square?
Which of the first 16 positive integers divide x? (Hint: keep **Problem 3** in mind).