

Lesson 1: Weighings, Logic and Geometric Constructions

Konstantin Miagkov

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Problem 1.

You are given a balance scale and a weight of 1 gram. Can you use them to measure out 1 kilogram of sugar by using the scale at most 10 times?

Problem 2.

There are 68 coins on the table, and any two coins weigh differently. Show how to determine the heaviest and the lightest coins using a balance scale at most 100 times.

We will explore the island of knights, knaves and spies, where there are three types of people: knights, who always tell the truth; knaves, who always lie; and spies, who can lie or tell the truth at will.

Problem 3.

On the island of knights, knaves and spies, you come across three people. One wears blue, one wears red, and one wears green. You know that one is a knight, one is a knave, and one is a spy. “Who is the spy?” you ask.

- The man wearing blue says, “That man in red is the spy.”
- The man wearing red says, “No, the man in green is the spy.”
- The man wearing green says, “No, the man in red is in fact the spy.”

Who is the spy? Who is the knight and who is the knave?

Problem 4.

There are 30 knights and knaves sitting at a round table. Each person has exactly one friend. Friend of a knight is always a knave, and friend of a knave is always a knight (friendship is mutual). You ask: “Is your friend sitting next to you?” and receive answers “Yes” from 15 people. How many other people could have possibly answered “Yes” as well?

Problem 5.

a) Given a segment AB on the plane, construct a point C such that ABC is an equilateral triangle using the ruler and compass.

b) Construct the midpoint of the segment AB .

Problem 6.

Given a point A and two rays out of it forming an angle α , construct the angle bisector of α .

Problem 7.

Let a_1, a_2, \dots be an infinite sequence of distinct positive integers, all of which are greater than 1. Show that there exist infinitely many i such that $a_i > i$.