

# Lesson 8: Quadratic Equations Mix

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Read all the problems before starting to solve any!

**Problem 1.**

Prove that if  $b$  is any real number and  $a \neq 0$ , then the quadratic equation  $ax^2 + bx - a = 0$  has two distinct real roots.

**Problem 2.**

$f(x) = x^2 + px + q$  is an even function. Find (with proof) the value of  $p$ .

**Problem 3.**

Points  $A, B, C$  lie on a circle of radius  $r$ , and  $AC = r$ . Find  $\angle ABC$ .

*Hint: Is there only one solution?*

**Problem 4.**

a) Find  $p$  and  $q$ , if 10 and  $-15$  are the roots of the quadratic equation  $x^2 + px + q$ .

b) Sum of squares of roots of a quadratic equation  $x^2 + px - 3$  is equal to 10. Find  $p$ .

**Problem 5.**

a) What is the minimum value of the function  $f(x) = x^2 + 10x - 13$ ?

b) What is its maximum value?

**Problem 6.**

Let  $O$  be the center of a circle and  $A$  be a point outside of it. Let  $AM$  and  $AN$  be tangents to the circle, and let  $B$  be the intersection of line  $AO$  with the circle furthest from  $A$ . Given that  $AN \parallel BM$ , find  $\angle OAM$  and prove that  $AMBN$  is a rhombus.