Lesson 6: Quadratic Equations V

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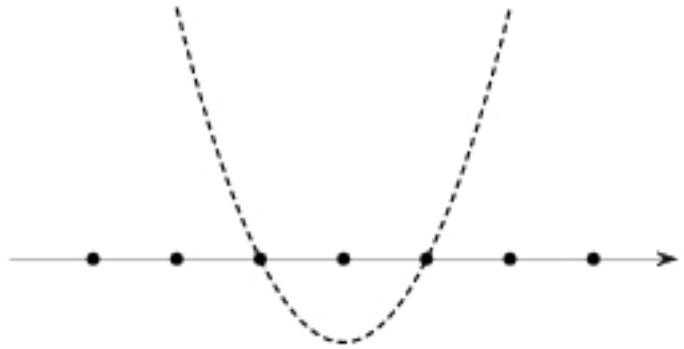
Problem 1.
Let $f(x) = ax^2 + bx + c$ be a quadratic function with $a > 0$, and set $h = -b/(2a)$. Show that the f is decreasing from $-\infty$ to h and increasing from h to $+\infty$. In other words, show that for all x, y such that $x < y < h$ we have $f(x) > f(y)$, and for all x, y with $h < x < y$ we get $f(x) < f(y)$. What happens when $a < 0$? *Hint: complete the square!*

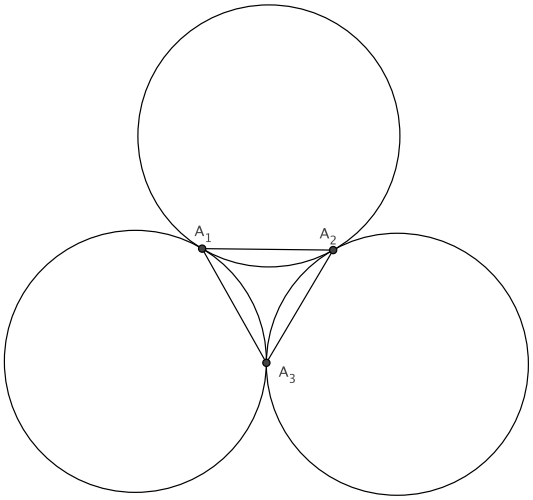
Problem 2.
Find all values of c for which the equation $x^2 - 4x + c = 0$ has two real roots whose sum of squares is 12.

Problem 3.
The picture below represents the graph of a quadratic equation f with the y-axis erased. Assume that f is monic ($a = 1$), and the consecutive points on the x-axis are 1 apart. Deduce the discriminant of f from this picture alone.



Problem 4.
Show that a quadratic equation f with two distinct real roots x_0, x_1 is an even function if and only if $x_0 = -x_1$.

Problem 5.
Three circles with the same radius 1 are pairwise externally tangent to each other. If A_1, A_2, A_3 are the three tangency points, find the angles and side lengths of the triangle $\triangle A_1A_2A_3$.



Problem 6.
In a triangle ABC we have $\angle ACB = 135°$. Let $ABMN$ be a square lying to the opposite side of C with respect to AB, and let O be the intersection of its diagonals. If $AM = 12$, find OC.