LAMC Beginners' Circle **October 6, 2013**

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A problem solving session

Problem 1 *Put the right sign, $>$, $<$, or $=$, between the fractions.*

$$\frac{2012}{2013} \quad \boxed{} \quad \frac{2013}{2014}$$

Problem 2 *The number 2^{1000} is written down in the decimal system. Would the number of its digits exceed 400? Why or why not?*

Problem 3 *The number 2^{1000} is written down in the binary system. What is the number of its digits? How many ones are there among them?*

Problem 4 *A ruler has three marks, zero, seven, and eleven inches.*

- *Is it possible to use the ruler for drawing a segment that is 8" long?*

- *How about a 5"-long segment?*

Problem 5 *Three houses A, B, and C, are built along a straight road.*

You are an engineer commissioned to find a place for a water well W so that the total distance from W to A, B, and C is the shortest possible. Where would you place the well?

Problem 6 *This time they have four houses, A, B, C, and D, located along a straight road.*

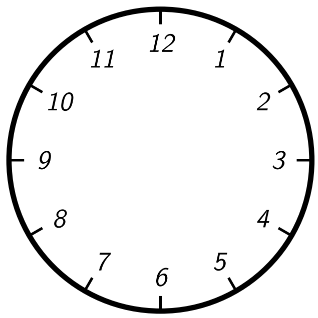
Where would you build the well?

Problem 7 *There are 100 schoolchildren living in the town of A. There are 50 schoolchildren living in the town of B. The towns are connected by a straight highway that is 3 miles long. The towns' councils decide to build one school for both towns. Where should they locate it so that the total distance of travel for all the 150 students summed together is minimal?*

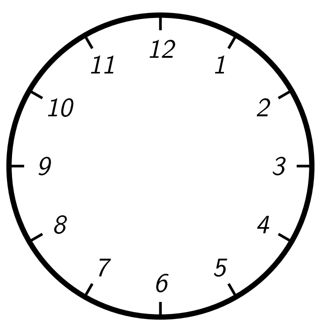
Problem 8 *At noon, the hour and minute hands of a clock point in the same direction. When would they point in the same direction next time?*

Problem 9 *How many times per day (24 hrs.) do the hour and minute hand point in the same direction?*

Problem 10 *Divide a clock face with two straight lines so that the sums of the numbers in each part are equal.*



Problem 11 *Divide a clock face into six parts so that each part contains two numbers and the six sums of two numbers are equal.*



Problem 12 *Find the following sum.*

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{99\times 100}=$$

Problem 13 *Prove that an integral number is divisible by three if and only if the sum of its digits is divisible by three.*

Problem 14 *Prove that an integral number is divisible by nine if and only if the sum of its digits is divisible by nine.*

Problem 15 *Oleg multiplied*

$$1 \times 2 \times 3 \times \ldots \times 100$$

and added up all the digits of the resulting number. Then he added up all the digits of the sum and then he proceeded doing so until he got down to a one-digit number. What was the one-digit number?

Problem 16 *The decimal representation of a number has one hundred digits zero, one hundred digits one, and one hundred digits two. Can the number be a perfect square? Why or why not?*

Problem 17 *Find the value of* 2×3 *in the place-value system where* $2 + 3 = 11$.

The following problem was communicated to me by one of our students, Ethan Kogan.

Problem 18 *Make a hundred using only four nines.*

Problem 19 *Prove the following polynomial identity.*

$$x^2 - y^2 = (x + y) \times (x - y)$$

Recall that a positive integer is called *prime* if it has only two positive integral factors, itself and one. For example, the number five is prime. The number six is not prime, since $6 = 3 \times 2$. A positive integer that is not prime is called *composite*.

Problem 20 *Prove that the number*

$$999,991$$

is not prime.

Problem 21 *Prove the following polynomial identity.*

$$x^3 + y^3 = (x + y) \times (x^2 - xy + y^2)$$

Problem 22 *Prove that the number* 1001_b *is composite for any base* b*.*

If you are finished solving all the above problems, please fill out the hexadecimal multiplication table below.

$\times$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f	10
0																	
1																	
2																	
3																	
4																	
5																	
6																	
7																	
8																	
9																	
a																	
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